Chapter 4

Experiment 2: Kinematics and Projectile Motion

Galileo is famous for several early mechanics experiments. In our first experiment, we reproduced the simple inclined plane to show that objects follow uniform acceleration in gravity. This conclusion about gravity is of course related to Galileo’s famous “leaning tower of Pisa” experiment.

Galileo also used an inclined plane to launch objects into the air to observe their projectile motion, or motion in two dimensions under the influence of gravity. In Galileo’s time, the recent invention of cannons prompted deeper understanding of projectile motion for improved warfare. In modern times, our interest in mechanics is a bit more general and fundamental. Here, we will use the same inclined plane apparatus from Experiment 1 to explore motion in two-dimensions; since this requires two position coordinates, we must use \((x(t), y(t))\) instead of \(s(t)\). Our approach will be different from Galileo’s, involving more sophisticated computer data acquisition, but the conclusions about mechanics will be the same: \textit{objects in motion in two dimensions with constant acceleration will follow a parabolic path}. In this experiment, we will investigate the vector acceleration in more detail than before. We will utilize computer analysis, and we will also note how an object will recoil from an elastic spring in preparation for our upcoming study of collisions. Just as the incline slows down gravity’s acceleration, the elastic band slows down the collision with the wooden frame so that we may study it in more detail.

4.1 Background: Kinematics

The mathematics of kinematics that was covered in the previous experiment is still relevant to this experiment, and you should refer to the material in Experiment 1 for a refresher. The definitions of position, velocity, and acceleration used here are all the same, except in this experiment we will need to distinguish between motion in more than one dimension. The concept of vectors, although relevant to Experiment 1, is now required. Since we have already shown that the acceleration due to gravity on an inclined plane is a constant along the plane, Equation (3.5) will be particularly important as well to describe motion under
gravity.

The primary insight that distinguishes dynamics in more than one dimension from the simpler one-dimensional case is that each direction is distinct and the governing equations can be written independently. This surprising structure is inherent in the vector form of Newton’s laws. Of course, motion in different directions can be coupled in many ways, but the laws of classical mechanics can always be written on separate orthogonal axes using vector components.

4.2 Hooke’s Law

In 1660 Robert Hooke realized that when a spring is compressed or stretched the effort needed grew larger as the amount the spring was deformed increased. Additionally, the effort (force) needed to compress the spring was opposite to the effort needed to stretch the spring. Hooke succinctly expressed this relation using the equation

\[ F_s = -kx \]  

(4.1)

where \( x \) is the signed amount that the spring is stretched and \( k \) is the constant of proportionality that characterizes each particular spring. The negative sign summarizes the fact that the spring tries to return to its undeformed shape and exerts a force, \( F_s \), itself to achieve this. Since this force will affect the puck’s motion while the puck is in contact with the elastic cord, we will find that the acceleration is not constant during these times. The student might recall that we omitted from consideration (and possibly did not record it at all) all of the puck’s motion after it contacted the wooden frame in the previous experiment. The puck’s motion was also greatly affected by that interaction and in the next few weeks we will begin to understand this more clearly. The elastic cord greatly slows down this reversal in motion so that we can investigate it. This strategy is not unlike using the inclined plane to slow down the effect of gravity itself.

4.3 Apparatus

**WARNING**

Whenever the push-button which activates the pulse generator is pressed, the puck should be on the white record paper and not on the carbon paper. The white paper should be on the carbon paper. If you touch the high voltage terminal and ground at the same time, you may get a shock – harmless but unpleasant! Always make sure that the ground clip is properly connected to the carbon sheet before activating the pulser.
The apparatus we use in this experiment is nearly identical to that used in Experiment 1. The primary difference is the addition of an elastic cord across the table (see Figure 4.1). Operation of the remaining components such as the ‘air hockey puck’, the Teledeltos paper, and the 60 Hz pulse generator are familiar.

4.4 Procedure

4.4.1 The puck’s positions

Consider the motion as taking place in one dimension. Consequently, the position is simply the displacement of the puck with respect to the origin of the coordinate system, or in other words, a measure of how far away (and in which direction) the puck is from the reference line (See Figure 4.2). The location of the origin and the orientation of the axes of a coordinate system is ours to choose as we find convenient. With the help of the inclined level, set the table at an angle of $6^\circ$ by adjusting the leveling screws until the air bubble is centered in the level and the level is squared in the frame. To avoid introducing an error due to parallax, position your eyeball directly above the level.

Helpful Tip

The lab technician has probably already oriented the incline, but you should check his work since your data depends upon it.

Place a sheet of carbon paper on the air table and connect it with the alligator clip to the pulse generator. Place a sheet of record paper on top of the carbon sheet, set the puck on the air table near the left side of the paper just barely touching the elastic cord, turn on the power to the pulse generator, and briefly press the sparker button. Move the puck near the right side of the paper just barely touching the cord and briefly press the sparker button. A straight line connecting these two dots will be our $x$-axis and will separate the motion into sections having a spring force and having no spring force.
Helpful Tip

Since these reference points are already on your paper, be sure not to disturb the record paper after you begin making these measurements until you have finished recording all of your data.

Turn on the pressurized air until the puck just starts to move freely. Open the valve another $5^\circ$ or so; if there are leaks in your air hose you might need to open the valve a little more still. Now your puck should rebound almost to the same height from which it was released. Make sure the puck does not hit the wood frame; release from a lower height or ask your teaching assistant to adjust your elastic cord if necessary. Allow the puck to slide down the incline from some position near the top of the paper, bounce off of the elastic cord, and come to rest again near the top of the paper. It will be helpful to give the puck a small horizontal velocity before releasing it so that the upward trajectory does not cover up the downward trajectory. After practicing a few times, activate the spark timer with the push button trigger as you release the puck; keep the button pressed until the puck comes to rest again near the top of the paper. Dots will be recorded every $\frac{1}{60}$th of a second but will appear on the underside of the paper.

**Figure 4.2:** An illustration of the preferred coordinate system for the analysis. It is convenient to separate the data into a class of points where the puck was accelerated by the elastic cord and a class where the puck was not. These coordinates separate those points by $y < 0$ and $y > 0$, respectively.

WARNING

Do not turn the air supply on all the way. It will damage the air hose. Only turn it on a little way. The puck will float well. Do not drop the puck. The table top is glass!

Remove the record paper and turn it over. Remember that flipping the paper interchanges the left and right sides. Use a straight edge to connect the points on the left and right of the paper where the puck first contacts the cord; call this line $y = 0$. Beginning with the first discernible individual dot at the beginning of the puck’s motion, circle every other dot (1, 3, 5, etc.). Measure the vertical distance from the $y = 0$ reference line to each of the circled dots in turn giving dots above the axis $+$ and those below the axis $−$. The distance we need is from the center of the line to the center of the dot when the ruler is perpendicular
to the line. Estimate the uncertainties in 1) positioning the ruler’s 0 at the center of the line and 2) reading the location of the dots’ centers. How confident are you that the ruler is perpendicular to the line? Will this affect your uncertainty estimate? If the cord was not carefully placed horizontally, will this affect your uncertainty estimate? Your ability to draw your reference line through the centers of the dots is also relevant. How might we estimate the uncertainties in our times?

We will enter these data into the Graphical Analysis (‘Ga3’) computer program from Vernier. The computer will do the tedious job of calculating the velocity and acceleration. The Ga3 program has a data window with columns for data entry. Double-click the first column header and label it as ‘t’ with units of seconds (‘s’). Calculate the entries by enabling the “Generate Values” option in the dialog box. Set the first time entry to 0, the interval to $\frac{1}{30} = 0.03333$, and the end time to about 2 seconds. Enter the position measurements into the second column labeling it as ‘y’ with the units of your measurements: m, cm, etc. As the data is entered the computer will plot the position as a function of time. The plot should resemble the trajectory of dots on the paper.

### 4.4.2 Calculate and plot the instantaneous velocity

This is the same task as in the first kinematics lab. You can focus on the vertical velocity so that it is a one-dimensional problem. We can calculate the instantaneous velocity by subtracting displacements of consecutive intervals ($\frac{1}{30}$ s). Actually, the result is the average velocity for the interval; however, such approximations are the only information available to experiments. There is always at least one instant within this interval (usually near the center) for which this average is the instantaneous velocity. Note that on the downward path consecutive positions decrease in value giving negative differences, and thus negative velocities. On the upward path consecutive positions get larger, giving positive differences and thus, positive velocities.

**Helpful Tip**

A common mistake is to forget that the velocity depends on the direction of the object’s motion. The puck slides down and yields negative velocity values. And then the puck bounces back up and generates positive values of velocity!

In the previous lab you took differences of position and divided by the time interval for each successive pair of position points. You can instruct the computer to perform this task by choosing “Data/New Calculated Column...” from the menu. Selecting this option will bring up a window in which you can specify the parameters of a new column. Title the column ‘v’ in units of velocity appropriate for your measurements (cm/s... etc.). In the “Equation:” edit control, select “Functions/delta” and “Variables (Columns)” position (y?). Next, divide by (/) “Functions/delta” and “Variables (Columns)” time (t?). Click “Done” and a new velocity column will be generated — actually this is average velocity for the $\frac{1}{30}$ s intervals,
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but this is the best we can do without calculus... Click the position graph and use the sizing handles to move the bottom up to about \( \frac{1}{3} \) of the screen; we want to display the velocity and the acceleration both under the position. “Insert/Graph” from the menu, adjust its height to about \( \frac{1}{3} \) of the screen, adjust its width to match the position graph, and use the dark border to position it under the position graph. If the \( y \)-axis is not already “Velocity”, click the \( y \)-axis label and choose velocity (\( v \)?). Similarly make sure the \( x \)-axis is time.

Now repeat this process to generate an acceleration column by dividing “delta(“\( v \)”)” by “delta(“\( t \)””). Don’t forget your column title and units and place acceleration on the third and bottom graph. Adjust the time axis scales and graph widths and positions so that the times are vertically aligned for all three graphs.

<table>
<thead>
<tr>
<th>Checkpoint</th>
</tr>
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<tbody>
<tr>
<td>Does the velocity decrease uniformly before the puck hits the spring? Does the velocity resume a uniform rate of decrease again after the puck leaves the spring?</td>
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4.4.3 Observe your graphs

Do you see any points on any of the graphs that seem out-of-place? If so, re-measure the positions near this time to be sure you have entered all of them correctly. With the time axis of each plot aligned (all minimum and maximum times are the same for the three axes) make some observations. Note the relative changes in each plot. For instance, when the puck slides down, it will do so at some constant acceleration (as in your first kinematics experiment). When it contacts the spring it is decelerated to a stop; i.e. the acceleration is opposite to the velocity or positive in this case. Note the relative sizes of the acceleration of gravity alone versus the spring force plus gravity. Is the spring force (acceleration) constant? Is it consistent with Hooke’s law \( (ma = F = -ky) \)? After the puck stops, the cord continues to accelerate the puck in the positive direction making the new velocities positive. The puck bounces back and loses contact with the spring. Finally, its acceleration depends only on gravity again and will again be a (negative) constant. Draw a box around these constant acceleration velocities before the spring and “Analyze/Linear Fit” to get a measured value of acceleration. If necessary right-click on the fit parameters box, “Linear Fit Options...”, and enable “Show Uncertainty” so that your measured acceleration is complete with its uncertainty. (It is also possible to draw a box around the constant acceleration points and “Analyze/Statistics” to find their mean and standard deviation.) To avoid mixing up your printout with those of your classmates, enter your name in the Text Window; you might also consider entering other information about your data such as the error in your measurements of \( y \) and \( t \). Obtain a hard copy of your table and plot for each of your lab notebooks (one for each member of your group) by using “File/Page Setup...” and “Landscape” orientation. Next “File/Print...”, click “OK”, and enter 2 (or 3) in the “number of copies to print” edit control.
4.4.4 Indicate where the puck was in contact with the spring

Indicate on the $y(t)$ graph, by a vertical line through all three graphs, the times at which the puck made contact with the spring ($y = 0$). Once the times when the puck first made contact with the spring and second left the spring are determined, note any worthwhile observations in your lab notebook.

Checkpoint

If you press the spring with your finger, does its reaction have the correct sign? Does the force get bigger as $y$ gets bigger? This is the minimum necessary for the cord to be consistent with Hooke’s law.

4.4.5 (Optional) Two-dimensional motion

Now that you have repeated your 1D analysis of the vertical motion with the spring, you can record the horizontal positions as well. Draw a $y$-axis perpendicular to the $x$-axis; you can utilize the 3-4-5 right triangle to do this more precisely. Now measure the $x$ locations of the same dots and enter them into the “$x$” column. Once again, the dots’ $x$ coordinates are signed. You can now plot a trajectory, which is a plot of $y$ vs. $x$. Except for scale, the trajectory should be identical to the dots on the paper.

Historical Aside

So far we have analyzed kinematics as position as a function of time. This is natural for our mathematical expressions in dynamics are functions of time. But, historically and practically, the trajectory shape (position vs. position), is often what one actually wants to know. In the case of Galileo and his contemporaries, cannon shot trajectories were of particular importance. Battleships of WWII had complex (and large) mechanical computers that performed calculations so that they could fire 2700 pound projectiles accurately toward a target 25 miles away.
4.4.6 (Optional) Hooke’s Law

We wish to observe evidence of Hooke’s law in our data. Soon we will learn about Newton’s second law of motion, the net force equals mass times acceleration \((F = ma)\). When the puck is stretching the elastic band, the force obeys Hooke’s law (see Equation (4.1)) so that

\[
ma = F = -ky
\]  

(4.2)

and the acceleration is proportional to the position. Insert a new graph of acceleration versus position and observe the result for \(y < 0\). Is the result consistent with Hooke’s law and Newton’s second law of motion?

4.5 Analysis

As in Experiment 1’s optional exercise, you should use computer software to fit your data and extract model parameters. These parameters will come with mathematical uncertainties that you can use to compare your measured accelerations of gravity to each other and/or to other measurements. (See Section 2.9.1.)

The most important part of this experiment is to examine the three curves for \(y(t)\), \(v(t)\), and \(a(t)\) all aligned on the same sheet of graph paper or computer plot, and to recognize the following relations predicted by our study of kinematics:

4.6 Lab Notebook Guidelines

Your grade will be based on two components: your in-class performance (including your lab notes) and your short written report communicating your work and its implications. Refer to Appendix E.

Your Lab Notebook should contain the following:

- A table of data collected in the lab (printed or hand written),
- Plots of the position, the velocity, and acceleration,
- One or two least-squares fit(s) for velocity,
- (Option) Graph of \(y\) vs. \(x\) depicting the trajectory,
- (Option) Graph of \(a\) vs. \(y\) illustrating Hooke’s law,
- Other measurements, ideas, and observations.

\(^1\)Thanks to Jonathan Trossman for this idea.
4.6.1 Calculus predictions for kinematics

Our study of kinematics predicts that your three graphs should have the following properties:

- The value of the puck coordinate $y(t)$
  - decreases until the bumper spring reverses the vertical velocity,
  - reaches a minimum value when the velocity is zero,
  - increases again until the end of the trajectory,
  - is minimum at the same time as acceleration is maximum.

- The velocity $v(t)$
  - increases in the negative direction at a uniform rate until after the puck comes into contact with the spring,
  - reaches maximum negative value (algebraic minimum) when acceleration is zero,
  - begins increasing (less and less negative and finally positive) after the minimum,
  - increases further becoming more and more positive, until just before the puck loses contact with the spring,
  - from there on the velocity decreases again (at a uniform rate only after the puck leaves the spring).

- The acceleration $a(t)$
  - is constant from the moment the puck is released until it comes in contact with the spring,
  - increases as the puck stretches the spring,
  - goes to zero as the velocity becomes minimum in value,
  - becomes positive and much larger than the acceleration due to gravity alone,
  - is maximum at the same time $y(t)$ is minimum,
  - goes to zero again as the velocity becomes maximum in value,
  - becomes constant again after the puck leaves the spring,

- Generally,
  - When the puck hits the bumper spring, – its trajectory $y(t)$ starts to flatten out,
  - its instantaneous velocity curve $v(t)$ does not immediately stop decreasing, and
  - its acceleration curve $a(t)$ does not immediately change sign.
  - When the puck is at the maximum compression point of the bumper spring, –
  - its trajectory has reached a minimum, – its velocity goes through zero, and – its acceleration is at a maximum.

Draw conclusions about our kinematic relations from these observations and note these observations in your discussion.
4.6.2 Thoughts on the written report

Refer to Appendix E. Some topics for discussion include:

- What physical relations have your data tested?
- Is Hooke’s law \( F = -ky \) feasible?
- How about Newton’s second law \( F = ma \)? Is acceleration greatest when applied force is greatest?
- What do your data say about calculus’ prediction that a function is extreme (maximum or minimum) when its derivative is zero?
- Recall that position’s derivative is velocity and that velocity’s derivative is acceleration. Both pairs of curves are predicted to have this behavior.
- Did you make any measurements worthy of reporting in your Conclusions?
- Is there an advantage to having the trajectory as a graph of \( y \) vs. \( x \) in addition to the dots on the record paper?
- Do you have a prediction for what a graph of \( x \) vs. \( t \) might look like?

Each of these topics should be addressed in 2-3 sections, but the particular sections vary with the topics. Refer to Appendix E.