Chapter 5

Experiment 3: Ohm’s ‘Law’

5.1 Introduction

When a potential difference is maintained between the contacts of an incandescent light bulb, the electric field forces charge to flow through the filament of the bulb. The filament resists the flow of charge, and the work done to force the charge through is converted to thermal energy, heating the filament to such a high temperature that it emits light. This light carries away energy. The temperature reached is such that the energy carried away by the light (and conducted away by the bulb holder) exactly balances the electric energy delivered to the filament. The flow of charge is described as an electric current and we would like to explore how the current through a particular material depends on the potential difference applied across the material object.

To examine this question adequately, we must first define quantitatively what is meant by “current” and “resistance”, as well as considering the physical mechanism that accounts for the electrical resistance of a metal. The current passing through a wire is defined by how much charge passes through a cross-sectional area, \( A \), of the wire per unit time, as shown in Figure 5.1. You should convince yourself that conservation of charge implies that the current is the same no matter what surface across the wire is used in defining the current. When the current changes in time, however, it is useful to consider the rate of charge crossing the area at each given instant, so that the current, \( I \), is defined more generally as

\[
I = \frac{dQ}{dt} \tag{5.1}
\]
CHAPTER 5: EXPERIMENT 3

where \( dQ \) is the charge passing through the area in some very brief time interval, \( dt \), when the observation is made. In the one dimensional case, a positive sign would be assigned to \( I \) when positive charge flows from left to right (positive \( x \)-direction) across the surface, a negative sign when it flows in the opposite direction. The unit of current is a Coulomb per second, which is given the special name “Ampere”, so that 1 Ampere is 1 Coulomb/second or, in official SI symbols, 1 A = 1 C/s.

We are primarily concerned with the current in a metallic conductor. In these systems, the electrons at the higher available energies are reasonably free to migrate from one location to another around the background of positively charged ions of the metal. The electrons moving in this way behave in many respects as if free. The background ions are mostly arranged in an orderly way on lattice sites. But because of the vibrational motion of the ions, and because of impurities and other unavoidable irregularities in the periodicity of the lattice, the electrons are constantly being scattered by the lattice of ions, interchanging energy and momentum with the heavier background ions. This randomizes the electronic motion and keeps the free electrons in thermal equilibrium with the lattice. Using the language of thermodynamics we would say that

\[
\frac{3}{2} M_L \langle v_L^2 \rangle = \frac{3}{2} k_B T = \frac{3}{2} m_e \langle v_e^2 \rangle,
\]

where \( M_L \) is the lattice molecular mass, \( v_L \) is its velocity, \( k_B \) is Boltzmann’s constant, \( T \) is the equilibrium temperature, \( m_e \) is the electron (effective) mass, and \( v_e \) is its velocity; the angular brackets indicate an average over particles and time. When an electric field \( E \) is applied along the conductor (see Figure 5.1), the field imposes a force \( F = -eE \) on the electrons (where \( e \) is the absolute value of the electron charge, \( e = 1.602 \times 10^{-19} \) C).

Scattering by the lattice prevents the electrons from accelerating indefinitely, but instead transforms the extra kinetic energy that the electrons acquire from the field into vibrational energy of the lattice. The conductor can become hot (as in the case of an electric stove), or even incandescent (as does the filament of a light bulb). The combined effect of the scattering and the applied field on the electrons is that their average velocity shows a small overall drift velocity, \( v_d \), (typically about 0.001 m/s) in the direction opposite to that of the applied electric field \( E \). This overall drift velocity of the electrons, superposed on their random thermal motion, is responsible for the electric current.

While the current in a metallic conductor is caused by the motion of electrons, there are systems that have moving positive charges or even both positive and negative charges moving simultaneously in opposite directions. A plasma is a gas of ionized particles in which the particles have had one or more electrons separated from a positive core or nucleus. The current in plasmas is due to the nuclei moving parallel to the electric field and the electrons moving opposite to the field. Conductive liquids, like the electrolyte in a car battery, have ionized atoms and/or ionized clusters of atoms. The electric current results from positive and negative ions moving in opposite directions. Semiconductors have electrons broken away from a chemical bond by thermal vibrations that then move about the crystal. The vibration leaves behind a positive ‘absence of an electron’ or hole that can be filled by borrowing an electron from a nearby bond. Semiconductor currents have holes moving parallel to the field and electrons moving opposite to the field. Hydrogen fuel cells have positive hydrogen ions moving into a membrane on one side and negative oxygen ions moving into the membrane.
on the other side (and in the opposite direction). The ions combine in the membrane to form neutral water.

### Historical Aside

Benjamin Franklin made significant contributions to the understanding of electricity by proposing a “single fluid theory” in which electrification by friction resulted from transferring particles of an “electrical fluid” (or “charge” from the modern viewpoint) from one object to the other. Objects with a deficiency of this fluid were negatively charged, and those with an excess would be positively charged. But, lacking our present knowledge of electrical phenomena, Franklin assigned a negative charge to the amber that had been rubbed with fur. This unfortunate choice was seen eventually to require assigning a negative charge to the electron, thus complicating the lives of future generations of physics students in their efforts to understand which way the charge ‘really’ flows in an electric circuit.

By convention, the direction of the electric current is taken to correspond to the flow of positive charges in the direction of the applied electric field from the higher to the lower electrical potential even when the actual charge carriers are negatively charged electrons which, in reality must move in the opposite direction (see Figure 5.1 and Figure 5.2). Sometimes, for emphasis, the term “conventional current” is used to distinguish the electric current defined in this way from the particle current motion of electrons. This seemingly artificial sign convention does not affect the validity of the equations based on it. The results are the same whether the current is taken to be that of positive charges flowing from a positive potential to a negative potential, or that of negative charge flowing from the negative potential to a positive potential. Indeed, close examination of the definition of current for the one-dimensional case, Equation (5.1), shows that a flow of negative charge to the left produces a positive current just as if positive charge really were flowing to the right.

### Checkpoint

What is an electric current?
Only the *Hall effect* can distinguish positive charge motion from negative charge motion mathematically. Later you will be able to understand how. When an electric field is constructed perpendicular to a magnetic field, the electric field accelerates positive and negative charges in opposite directions. Because of their opposite movement and their opposite charges, both kinds of charged particles collect on the *same* side of the object. This side becomes positively charged by motion of positive charge carriers and negatively charged by negative carriers. We can tell the sign of the dominant charge carriers by noting the potential on this surface.

### 5.1.1 The Electromotive Force

Maintaining a steady current in a wire requires a source of electrical energy, such as an electric battery or electric generator (see Figure 5.2). Chemical energy in the battery or mechanical energy in the case of the generator, is converted into electrical energy by doing work on the charges passing through. This “electromotive force” (or “emf”) is the work done per unit charge by the battery or generator to move charge from lower to higher potential. The units of emf are Volts (V). For a battery, the emf is also equal to the voltage across the battery terminals when nothing is connected to the battery terminals and no significant current flows. Note that the customary term “electromotive force” is somewhat misleading since the emf is an electric potential difference and not a force.

**Checkpoint**

What is an emf?

### 5.1.2 Resistance and Ohm’s Law

Early in the nineteenth century a German mathematician and physicist named Georg Simon Ohm discovered that, as long as the temperature was kept constant the magnitude of the current in a metal is proportional to the applied voltage as shown in Figure 5.3. This relationship is now known as Ohm’s law. Ohm defined a property of the conductor that he called the resistance, \( R \), to be the proportionality constant in this law,

\[
R = \frac{V}{I} \quad \text{or} \quad V = IR. \tag{5.2}
\]

Resistance resists the flow of charge; a larger resistance requires a larger emf or potential difference, \( V \), to result in the same current, \( I \). The units of resistance is the Ohm (1 Ω = 1 V/A) in Georg’s honor.

The resistance of a particular conductor or resistor depends on the physical dimensions of the resistor material and the *resistivity* property of the material itself. If a resistor has
length, \( L \), uniform cross-sectional area, \( A \), and resistivity, \( \rho \), then its resistance will be

\[ R = \rho \frac{L}{A}. \]  

(5.3)

The properties of materials, including their resistivity, tend to change as thermodynamic attributes such as temperature changes. In the particular case of resistivity, the change with temperature is fairly linear over a wide range of temperatures for most materials. The slope of this line is the temperature coefficient of resistivity, \( \alpha \), and also is a property of materials. We model the temperature dependence of resistivity using

\[ \rho(T) = \rho_0 [1 + \alpha (T - T_0)] \]  

(5.4)

where \( \rho_0 \) is the measured resistivity at temperature \( T_0 \). The resistance of most conductive materials (and all metals in particular) increases with temperature increases so that \( \alpha > 0 \) for most conductive materials. The resistivity of insulators and semiconductors, on the other hand, tends to decrease with increased temperature and \( \alpha < 0 \).

Since vibrations of the ions around which the electrons move typically produce most of the scattering that impedes the response of the electrons to an applied field, and since the amplitude of these vibrations increases with temperature, the resistance of most solids tends to increase with temperature as shown in Figure 5.3. But thermal vibrations can also break electrons free of chemical bonds and thus increase the number of moving charges and the current. In insulators and semiconductors that have very few free charges, freeing charges in this way decreases resistance more than the increased scattering increases it.

**Checkpoint**

Which way does the current flow when the applied electric field is from left to right along a wire? Which way do the electrons flow?

**Checkpoint**

State Ohm’s Law.
Anything that has electrical resistance as its primary electrical response is termed a resistor. In a circuit diagram, the symbol for a resistor is a zigzag line (see Figure 5.4). Everything (except superconductors) has some electrical resistance and we include a resistor symbol in its model when we wish to emphasize this fact. The resistors used in most applications are “carbon composition” resistors - small cylinders with wire leads at each end, constructed of powdered graphite in clay. The resistivity can be controlled using the ratio of graphite to clay. Most resistors are color-coded with a series of colored bands on them to indicate their designed resistance. Figure 5.4 and Table 5.1 detail how this color code is defined and used.

The first two bands define the significant digits or mantissa of the scientific notation. The ten possible colors correspond to the ten decimal digits 0-9. The third band indicates the exponent of the scientific notation or the number of zeros that need to be added after the two digits. In addition to the normal ten digits, the exponent band can also be silver or gold to allow resistances less than $10\,\Omega$. A silver exponent band means move the decimal point two places to the left ($0.10\,\Omega$ - $0.91\,\Omega$) and a gold band means move it one place to the left ($1.0\,\Omega$ - $9.1\,\Omega$). The last band, which is gold, silver, or simply absent, denotes

**Figure 5.4:** *On the left we have the schematic symbols for select circuit elements. On the right we illustrate the definition and usage of the resistor color code.*
the manufacturer’s tolerance in achieving the indicated value of resistance. Nothing can be made arbitrarily well. The molds the manufacturer uses to form the composite carbon and clay have error tolerances in their area and length. The composite that fills the molds has tolerances for the ratio of graphite to clay as well as for any impurities that cannot be eliminated from the graphite and clay. The manufacturer has tolerances for how much composite material gets packed into each mold. But all together the manufacturer is confident that his product will maintain its resistance within the specified tolerance for all temperatures and humidities that do not damage the device. The fourth band summarizes the specified tolerance: gold $\Leftrightarrow 5\%$, silver $\Leftrightarrow 10\%$, and no band means 20\%. If a fifth band is present, it specifies the product reliability or failure rate. An example of using this coding scheme is shown and explained in Figure 5.4.

The work done on a charge, $q$, by an electric field, $E$, is

$$W = \int_a^b F \cdot dr = q \int_a^b E \cdot dr = qV_{ab}. \quad (5.5)$$

To compute the power expended by the electric field in moving a current through a resistor, we need the rate that work is performed. Since the potential difference in this case is independent of time,

$$P = \frac{dW}{dt} = \frac{dq}{dt} V = IV. \quad (5.6)$$

We can use Ohm’s law to substitute for $I$ or for $V$ to find three equivalent forms of Joule’s law of electrical heating,

$$P = IV = I^2R = \frac{V^2}{R}, \quad (5.7)$$

and dissipated power has units of Volt-Amperes or Watts after James Watt who invented the steam engine and several thermodynamics facts. The units of Volts and Amperes have been defined in a way that relates electrical and mechanical quantities, because a Volt is an amount of work per Coulomb, so that

$$1 \text{ Volt-Coulomb} = 1 \text{ Volt-Ampere-second} = 1 \text{ Watt-second} = 1 \text{ Joule} = 1 \text{ Newton-meter}.$$
**CHAPTER 5: EXPERIMENT 3**

**Electrical Circuits and Circuit Diagrams**

We illustrate circuits using *schematic diagrams* whose components are depicted with symbols. These symbols play the same role in our study of circuits as the alphabet plays in written language or the ten numerals play in our number system. Figure 5.5 is one example of the schematic diagram. We might contrast this with the ‘connection diagram’ in Figure 5.6 or the apparatus’ photograph on the lab’s website. The schematic diagram contains all information needed to construct the circuit from arbitrary components. Many different corporations manufacture resistors, voltmeters, ammeters, and power supplies and it is thus unlikely that our readers each have exactly our combination; however, if he chooses he can reproduce our work using the components that he has in his lab.

**5.1.3 Equipment**

The plug-in boards we will use in this experiment allow you to assemble a circuit quickly from component parts. In this lab we will assemble a simple circuit with a power source (emf) and a single resistor. We will include in the circuit a means of measuring the voltage of the emf and the corresponding current that this emf drives through the resistance as we vary the emf. We will do this experiment for each one of three resistances with values of 1 kΩ, 470 Ω and 100 Ω, for a light bulb, and for a light emitting diode (LED). To prepare for the lab you should calculate the currents through resistances of the values above for a supply voltage of 6 Volts maximum. The resistances you will use are rated for 2.0 Watts. Also, compute the power each resistor will dissipate and check to make sure this power rating is not exceeded for any of the resistances you will be using BEFORE you power up the circuit for real.

Pasco’s 850 Universal Interface will provide a variable voltage power source that you can control from their Capstone computer program. The voltage output leads can be located on the right top of the 850 Interface box front panel. Connect the positive and negative terminals to the appropriate sockets on the plug-in board. The negative output is indicated with a ground symbol (see Figure 5.4) and the positive output is indicated with a sine wave. Construct a simple circuit using the 100 Ω resistor as shown in Figure 5.6.

The voltage and current will be measured using Pasco’s 850 Universal Interface and Capstone program. Connect the voltage leads to input A on the 850 Interface box. Apply the leads to the ends of the circuit element to be tested, as shown in Figure 5.6 for the case...
Figure 5.6: Sketch of an electric circuit constructed on our plug-in breadboards. The circuit consists of the Interface’s output providing an emf, an ammeter to allow the current to be measured, and a resistor to limit the flow of current. External to the circuit and yet essential is the voltage sensor in parallel with the resistor.

of a resistor. Execute Capstone’s setup file from the lab’s web page at

http://groups.physics.northwestern.edu/lab/ohms-law.html

You can begin and stop collecting data by clicking the “Record” button at the bottom left. The button then changes into a “Stop” button so that clicking it again ends the collection run.

Click the “Signal Generator” at the left to bring up the power supply control. It should already be configured to supply a constant potential difference and increasing or decreasing the output level will vary the particular potential difference generated. Set the emf you desire and click “Record”. Verify that both the resistor current and the resistor voltage are constant and click “Stop”. Small variations in the current and voltage readings suggest the 850’s uncertainties in these measurements. Record the voltage and current in a data table
or enter them directly into Vernier Software’s Graphical Analysis 3.4 (Ga3); a suitable setup for Ga3 is also supplied on the website. Estimate the uncertainties and enter these into your data table as well.

Now choose another applied voltage and “Record”, observe, and “Stop” again to generate another \((I, V)\) data point for your data table (and graph). Select about 10 different applied voltages between -5V and +5V for each component. Do your data points lie along a line in your graph? If they do, draw a box around the data points, “Analyze/Curve Fit…”, and choose “Ohm’s Law” from near the bottom of the list. “Try Fit” and “Done” to fit them to Ohm’s law. If they do not lie along a line, it makes no sense to model them as a line; can you think of another model that might work? If not, just display the data points. If your fit parameters do not include the uncertainties, right-click the box, “Properties…”, and select “Show Uncertainties”. Print the data table and the graph for your report.

**Helpful Tip**

If you want to use a Word processor, you can select the table and then the graph and copy each in turn directly into the Word report. Don’t forget to generate a label (Caption) so that you can talk about it in your main text.

A suitable Word template is also provided on the website to get you started.

**Checkpoint**

Why must the voltage be read directly across the material being tested rather than using the voltage reading of the source?

**Checkpoint**

Why must the Ammeter interrupt the circuit and not be placed in parallel with the element through which the current is being measured?

Repeat the experiment for the other two resistors. Repeat for the incandescent lamp; is the lamp data linear? If not, it would be counterproductive to fit it to a line. Plot current on the \(x\)-axis and voltage on the \(y\)-axis by clicking on the axis labels in turn. Does the graph have odd symmetry? (Is \(f(-x) = -f(x)\)?) If so, the function has only odd powers and we can model it using

\[
V(I) = R_0 I + R_2 I^3 + \ldots
\]

\[(5.8)\]

Data/Sort Data using current. Drag a box around your data points and Analyze/Curve Fit… Choose the “Odd Cubic” model near the bottom of the list.

Why is the lamp data not linear? Have you made observations to support this? Note
them in your Data. Be as detailed in your explanation as you know how to be. Offer all evidence in support that you remember observing. Feel free to gather more data so that you can pay closer attention to details.

Repeat once more using the LED. Is the LED’s data linear? Does the LED’s data have odd symmetry? Even symmetry? No symmetry at all? Is there any reason why this data should have more error than the data from the other samples (resistors or lamp)?

**Checkpoint**

What is meant by the term “ohmic material”?

### 5.2 Analysis

Calculate the Difference between the resistance measured from your graph slopes and the values specified by the resistor manufacturer,

\[
\Delta = |R_{\text{manufacturer}} - R_{\text{measured}}|.
\]

(5.9)

It might be necessary for you to convert the units on one of these two measurements before you can subtract them. Since you measured two resistors, you will compute two Differences. Compare these differences with the tolerances predicted by the manufacturer. What other subtle experimental errors are present in your measurements? Might some of these be large enough to explain any additional disagreement?

Discuss the lamp’s data. Is current directly proportional to voltage as Ohm predicted? Might we correct for resistance changes to make the lamp’s voltage proportional to current at each data point? What is the cause of these resistance changes? Keep in mind that the lamp’s filament is the metal tungsten (W) and describe in detail how increasing voltage can result in resistance increases.

Discuss the LED’s data. Is current directly proportional to voltage as Ohm predicted? Do you even get the same absolute value of current for positive and negative voltages? Does the LED get hot? Light it and touch it to be sure. Do LEDs obey Ohm’s law? Is Ohm’s “law” a law or does it have exceptions? Can you prove your answer?

### 5.3 Conclusions

How well do resistors obey Ohm’s law? Might Ohm’s law be useful to electrical engineers? Consider including your measured resistance values and units. Discuss what your lamp says about Ohm’s law and correcting resistance for temperature changes. Restate your \(V(I)\) model for your lamp, define all symbols, and give measured values and units for \(R_0\) and \(R_2\).
Discuss what your LED says about Ohm’s law. Is Ohm’s law a law?

Always communicate with complete sentences; “Yes” is a very poor answer for these suggested discussion topics and equations are not sentences but may be used as nouns.