Chapter 9

Experiment 7: Electromagnetic Oscillations

9.1 Introduction

The goal of this lab is to examine electromagnetic oscillations in an alternating current (AC) circuit of the kind shown in Figure 9.1. The circuit differs from that in the $RC$ circuit experiment by including the solenoid coil of wire, labeled “L.” When the switch is moved from position 1 to position 2, the charged capacitor discharges; however, the voltage across the capacitor does not now simply decay to zero as we saw for the $RC$ circuit. Since an inductor stores energy in its magnetic field, this circuit’s capacitor voltage oscillates between positive and negative values as time passes; we saw similar behavior when we studied the pendulum. Electrical resistance dissipates the capacitor’s energy ($P = V^2/R$) so that its voltage slowly goes to zero ($U_C = CV^2/2$). Since even good wires and oxidized metal in contacts have some resistance, the amplitude of the oscillations seen in this $RLC$ circuit will also slowly go to zero.

![Figure 9.1: A schematic diagram of our RLC circuit. The resistance is not explicitly added but some of our other components have resistance. The battery and switch will be replaced by a square-wave generator.]

![Figure 9.2: A plot of an exponentially decaying oscillation. Each time the switch flips, the RLC circuit responds with damped oscillations.]

In fact, there is a strong analogy between mechanical oscillations (the pendulum) and
electromagnetic oscillations. The same differential equation of motion describes both; only
the constants in the equation \((1/l \Leftrightarrow L, g \Leftrightarrow C, b \Leftrightarrow R)\) and the changing function
\((\theta(t) \Leftrightarrow V_c(t))\) are different. These physical characteristics of the mechanical oscillator
therefore correspond to their specific electromagnetic counterparts in the AC circuit studied
here. The same equations that describe the oscillation of mechanical quantities also describe
the oscillation of electromagnetic quantities and many results apply equally for mechanical
oscillators and electromagnetic oscillators. To explain the oscillatory behavior of the circuit
in Figure 9.1 and the close analogy between mechanical and electromagnetic oscillations, we
first discuss Faraday’s law, inductors, and alternating current circuits.

### Historical Aside

Other oscillators include a mass on a spring that leads to tuning forks, bells, and guitar
strings. An electron in an atom in the process of emitting a photon is an oscillator.
Atoms in solids oscillate about discrete lattice points. Because these oscillators all act
alike, we once utilized “analog computers” to simulate these other phenomena using
electric circuits at a much lower cost.

Every electric current generates a magnetic field. The magnetic field around the
moving charge can be visualized in terms of

### Checkpoint

What physical quantity in an AC circuit plays the same role that frictional
forces play in a mechanical oscillator? Why?

magnetic lines similar to the electric lines of force for the electric field. At each point
the direction of the straight line tangent to a magnetic field line gives the direction of the
magnetic field in complete analogy to the electric lines of force. Magnetic field lines
are not lines of force since, as we saw last
week, magnetic forces are perpendicular to
the fields not parallel to them.

The solenoid used as an inductor in this experiment (component \(L\) of Figure 9.1)
consists of a long insulated wire wound
around a cylinder with a constant number of

Figure 9.3: A sketch of the magnetic field
lines formed by current flowing in a solenoidal
coil of wire. As shown, the field is formed
by current flowing upward on the side of the
coil toward us and downward on the side away
from us. The right-hand-rule predicts the
north pole on the left.
turns per unit length of the cylinder. The current, $I$, through the wire produces a magnetic field as illustrated in Figure 9.3. To examine the role of the solenoid, consider more generally any area, $A$, enclosed by a closed path $P$ in space, as illustrated in Figure 9.4. Then with the magnetic field at each point in the region we can define the flux

$$\Phi = \int_{\text{area}} \mathbf{B} \cdot d\mathbf{a} \quad (9.1)$$

as essentially the number of lines of the $\mathbf{B}$ field (or the “amount” of $\mathbf{B}$ field) passing through the area. If the lines of the magnetic field are perpendicular to the area, $A$, then the flux $\Phi$ is simply the product of the magnetic field times the area. The dimensions of the magnetic flux are Webers (or Wb) with

$$1 \text{ Wb} = 1 \text{ Volt-second}.$$

Figure 9.4: An illustration of magnetic flux contained by a closed path in space. The path encloses an area, $A$, that can be divided into infinitesimal pieces, $d\mathbf{a}$

Michael Faraday (in England) and Joseph Henry (in the United States) independently discovered that a change in flux induces an electromotive force (emf). Henry made the discovery earlier but published it later than Faraday. Specifically, a changing flux through the area enclosed by the path in Figure 9.4 produces an emf around the closed path. The emf is defined as the work per unit test charge that the electric field would do on a small positive charge moved around the path, $P$, and is equal in magnitude to the rate of change of the flux through the circuit,

$$\mathcal{E} = -\frac{d\Phi}{dt}. \quad (9.2)$$

The physical relation given by Equation (9.2) is usually referred to in physics as Faraday’s law. The minus sign is required by Lenz’s law, and indicates that the induced emf is always in a direction that would produce a current whose magnetic field opposes the change in flux (or tends to keep the flux constant).

Suppose the loop defining the area, $A$, is made of copper wire with ohmic resistance $R$. Ohm’s law means that any emf induced in the loop will cause a current $I = \mathcal{E}/R$ to flow through the loop. A coil of $N$ turns with the same changing flux through each would be equivalent to $N$ single loops in series, each with the same emf thereby producing the emf,

$$\mathcal{E} = -N \frac{d\Phi}{dt}. \quad (9.3)$$

As the capacitor shown in Figure 9.1 discharges, the current would simply decrease exponen-
tially if the solenoid had no effect. But the increasing current changes the flux through the
solenoid and thereby induces an emf acting on the solenoid itself (referred to as a “back emf”)
that slows the rate that the current increases. Once the capacitor has discharged completely,
the inductor has built up a substantial current and a substantial magnetic flux. This current
will begin to charge up the capacitor with the opposite (negative) polarity and this negative
emf will now begin to decrease the inductor’s current. As the capacitor’s charge (and emf)
increases in the negative direction, the rate of current change also becomes more negative
until it finally becomes zero once again. Now the magnetic flux is zero, but the capacitor
has a substantial negative charge and voltage. This negative emf will, similarly, cause a
negative current and flux to increase in absolute value until the capacitor has expended its
electric energy while building magnetic energy. This magnetic energy will once again charge
the capacitor with positive voltage completing the cycle. This positive capacitor voltage will
then begin a new and similar cycle that will repeat endlessly unless the energy is dissipated
instead of being swapped back and forth between electric energy and magnetic energy.

For a pendulum we found that gravitational potential energy \( U_g = mgL(1 - \cos \theta) \)
slowly became kinetic energy in the pendulum bob \( K = \frac{1}{2} mv^2 \) until \( \theta = 0 \) and \( U_g = 0 \). At
this point the bob had substantial momentum that forced the pendulum past the equilibrium
\( \theta < 0 \) and \( U_g > 0 \) until it had built up substantial gravitational potential energy. This
gravitational potential energy and kinetic energy was swapped back and forth until the
friction in the pivot and air resistance dissipated the energy. When we studied the pendulum,
we included no mechanism to dissipate the energy so the prediction was that the oscillations
would be persistent; including a damping term \( \tau_\omega = -b \frac{d\theta}{dt} \) in the pendulum’s torques would
have made that motion equivalent to this.

To examine the circuit more quantitatively, note that for a coil of \( N \) turns of wire and
each with flux, \( \Phi \), the total flux, \( N\Phi \), through the coil must be proportional to the current
through the wire,

\[
N\Phi = LI \tag{9.4}
\]

The proportionality constant in this relation is the self-inductance \( L \) given by

\[
L = N \frac{\Phi}{I} \tag{9.5}
\]

The inductance is a characteristic property of the inductor determined by its geometry (its
shape, size, number of windings, material near the windings, and arrangement of windings),
just as the capacitance of a capacitor depends on the geometry of its plates and on whatever
separates them from each other. The units of \( L \) are Volt-second/Ampere with

\[
1 \text{ V-s/A}=1 \text{ Henry}
\]

(with the symbol for Henry being \( H \) not to be confused with Hertz, Hz).
CHAPTER 9: EXPERIMENT 7

**Checkpoint**

What is magnetic flux? What is Faraday’s law? Who first discovered it?

Because Equation (9.5) relates the flux through the circuit to the current at each instant of time, the time rate of change of the two sides of Equation (9.5) must also be equal. Since the left side of the equation is the total flux linkage, \( N \Phi \), its rate of change is the induced emf, while the rate of change of the right hand side is \( L \) times the rate at which the current changes at each instant. Therefore the back emf is equal to \( L \) multiplied by the rate of change of the current; equivalently, we simply differentiate both sides of Equation (9.4),

\[
\mathcal{E} = -N \frac{d\Phi}{dt} = -L \frac{dI}{dt}. \tag{9.6}
\]

**Helpful Tip**

Note that we use \( \mathcal{E} \) for the emf induced by changing flux in the coil. When analyzing a circuit containing an inductor, we need \( V_L \) across the inductor that causes the flux to change. These action-reaction effects have opposite signs.

**Checkpoint**

What physical features determine the inductance of a solenoid?

### 9.1.1 Energy Considerations

As a charge, \( dQ \), flows through the circuit, it gains energy \( V \, dQ \), where \( V = L \, dI / dt \). Thus, the energy lost by the charge (which is the energy given to the inductor) is

\[
dU_L = V \, dQ = \left( L \frac{dI}{dt} \right) \, dQ = L \, dI \left( \frac{dQ}{dt} \right) = LI \, dI. \tag{9.7}
\]

If we start with zero current, and build up to a current \( I_0 \), the energy stored in the inductor is

\[
U_L = \int_{U_L(0)}^{U_L(I)} dU = L \int_0^I I' \, dI' = \frac{L}{2} \left[ I^2 \right]_0^I = \frac{1}{2} LI^2 \tag{9.8}
\]

This energy is stored in the form of the magnetic field. When the switch, \( S \), in Figure 9.1 is in position 1, the capacitor becomes charged. Eventually, the capacitor has the full voltage \( V_0 \) across it and has energy \( CV_0^2 / 2 \) stored in the electric field between its plates. Once
we set the switch to position 2 at $t = 0$, current flows through the inductor building up a magnetic field. As the voltage oscillates, the energy oscillates between being magnetic energy of the inductor and electric field energy of the capacitor. Some of the energy is lost to the environment as heat because of the ohmic resistance of the circuit and any other conductors that get linked by the flux lines of the inductor. These losses decrease the maximum of the energy $CV^2/2$ stored in the capacitor (and $LI^2/2$ stored in the inductor) in each successive cycle. The amplitude as measured by the maximum of the voltage, $V_C$, across the capacitor therefore decreases from each cycle to the next.

**Checkpoint**

In terms of energy, when can a system oscillate?

**Checkpoint**

What is the energy in an inductor in terms of quantities such as charge, current, voltage?

### 9.2 Damped Oscillations in RLC Circuits

We next need to examine the precise time dependence of the voltage $V_C(t)$ and of the current $I(t)$ in a circuit such as that illustrated in Figure 9.1. Although the circuit includes a capacitor, an inductor, and a resistor, the behavior of the circuit is determined by the total resistance $R = R_L + R_{\text{misc}}$, total inductance $L$, and total capacitance $C$ of the entire circuit, rather than the value added explicitly by the components. Each component of the circuit contributes to these three physical aspects of the circuit; the resistor, for example, also has a slight capacitance and inductance like the inductor has resistance in its wire and capacitance between its windings. Since the resistance of the short wires is fairly negligible, the resistance measured from one side of the capacitor to the other in Figure 9.1 is seen to be $R = R_L + R_{\text{misc}}$. The relation between voltage and current for these three elements are summarized in Table 9.1, together with the SI symbol and the expressions for the energy associated with each physical quantity. In any closed circuit the sum of the voltages across the components must be zero, so that

$$V_L + V_R + V_C = 0. \quad (9.9)$$

Based on the expressions for the voltage differences in Table 9.1,

$$L \frac{dI}{dt} + IR + V_C = 0; \quad (9.10)$$
**Table 9.1:** A list of our circuit elements and information about them.

<table>
<thead>
<tr>
<th>Element</th>
<th>Symbol</th>
<th>Schematic</th>
<th>Voltage</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>$R$</td>
<td>$\mathcal{R}$</td>
<td>$V_R = IR = RC \frac{dV_C}{dt}$</td>
<td>$P = FR$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$L$</td>
<td>$\mathcal{L}$</td>
<td>$V_L = L \frac{dI}{dt} = LC \frac{d^2V_C}{dt^2}$</td>
<td>$U_L = \frac{1}{2} L I^2$</td>
</tr>
<tr>
<td>Capacitor</td>
<td>$C$</td>
<td>$\mathcal{C}$</td>
<td>$Q = CV_C$; $I = C \frac{dV_C}{dt}$</td>
<td>$U_C = \frac{1}{2} CV^2$</td>
</tr>
<tr>
<td>Battery</td>
<td>$V$</td>
<td>$\mathcal{F}$</td>
<td>$\frac{d^2V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{V_C(t)}{LC} = 0$</td>
<td></td>
</tr>
<tr>
<td>Square-wave</td>
<td>$V_0$</td>
<td>$\mathcal{W}$</td>
<td>$\frac{d^2V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{V_C(t)}{LC} = 0$</td>
<td></td>
</tr>
</tbody>
</table>

but the current is due to charge flowing onto (or off of) the capacitor plates, so

$$I = \frac{dQ}{dt} = C \frac{dV_C}{dt}, \quad \frac{dI}{dt} = C' \frac{d^2V_C}{dt^2}, \quad (9.11)$$

and

$$LC \frac{d^2V_C}{dt^2} + RC \frac{dV_C}{dt} + V_C(t) = 0$$

$$\frac{d^2V_C}{dt^2} + \frac{R}{L} \frac{dV_C}{dt} + \frac{V_C(t)}{LC} = 0. \quad (9.12)$$

This equation occurs so often in math and physics that the mathematicians have solved it in general. We can simply look it up in books containing tables of integrals and solutions to differential equations. To make finding this particular solution easier, differential equations are classified. This one has a second derivative so it is second order. The terms having the function, $V_C$, in them are multiplied by constants and not functions of time, so its class has constant coefficients. Since the function on the right is 0, this class is homogeneous. We need to look for solutions of **homogeneous second order differential equations with constant coefficients**. We will find that

$$\frac{d^2f}{dt^2} + 2\gamma \frac{df}{dt} + \omega_0^2 f(t) = 0 \quad (9.13)$$

has the solution

$$f(t) = f_0 e^{-\gamma t} \cos(\omega' t + \varphi) \quad (9.14)$$

where $\omega' = \sqrt{\omega_0^2 - \gamma^2}$ and $f_0$ and $\varphi$, are constants of integration that can be found from initial conditions. If we compare Equation (9.12) to Equation (9.13), we can identify

$$2\gamma = \frac{R}{L}, \quad \omega_0 = \frac{1}{LC}, \quad \text{and} \quad f(t) = V_C(t) \quad (9.15)$$

119
so
\[ \gamma = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}, \quad \omega' = \sqrt{\omega_0^2 - \gamma^2}, \quad \text{and} \quad V_C(t) = V_0 e^{-\gamma t} \cos(\omega't + \varphi). \] (9.16)

You can convince yourself that Equation (9.14) gives the correct solution by substituting into Equation (9.12).

The function \( V_C(t) \) is that shown in Figure 9.2 and Figure 9.5. It is the product of an oscillatory term and an exponential decay function that damps the amplitude of the oscillation as the time increases. The period of the cosine (and sine) function is \( 2\pi \) radians so we can find the period, \( T \), of the oscillations using

\[ 2\pi = \omega' T \quad \text{and} \quad T = \frac{2\pi}{\omega'}. \] (9.17)

The damping factor, \textit{gamma}, does not affect the period, \( T \), of the oscillatory term very much. The transfer of energy back and forth from the capacitor to the inductor is illustrated in Figure 9.5. In this figure the current, \( I = \frac{dQ}{dt} \), is also plotted.

At the time marked 1 in Figure 9.5 all the energy is in the electric field of the fully charged capacitor. A quarter cycle later at 2, the capacitor is discharged and nearly all this energy is found in the magnetic field of the coil. As the oscillation continues, the circuit resistance converts electromagnetic energy into thermal energy and the amplitude decreases.

\[ \text{Figure 9.5: Sketches of capacitor voltage and circuit current showing how electric energy and magnetic energy swap back and forth between the capacitor and the inductor.} \]

\[ \text{Checkpoint} \]

Which of these circuits – \( RC, RL, LC, RLC \) – can produce oscillations? Explain.
9.2.1 Observing damped oscillations in a $RLC$ circuit

We connect a capacitor, a resistor, and an inductor in series on a plug-in breadboard as shown in Figure 9.6. The set-up differs from the schematic diagram in Figure 9.1 by including connections to the computer-based oscilloscope to monitor the time dependence of the input and capacitor voltages. Also, just as in the case of the RC circuit experiment, the switch in Figure 9.1 is replaced by the square-wave output of the function generator. If you have forgotten how to use the computer-based oscilloscope, it would be a good idea to read the section of the fifth lab write up in which its operation is described.

Table 9.2: A list of the units of the three main quantities characterizing the circuit.

<table>
<thead>
<tr>
<th>Name</th>
<th>symbol</th>
<th>SI Unit Name</th>
<th>Unit</th>
<th>Unit Definition</th>
<th>Short</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance</td>
<td>$R$</td>
<td>Ohm</td>
<td>$\Omega$</td>
<td>Volts per Ampere</td>
<td>V/A</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L$</td>
<td>Henry</td>
<td>$H$</td>
<td>Volt-second per Ampere</td>
<td>Vs/A</td>
</tr>
<tr>
<td>Capacitance</td>
<td>$C$</td>
<td>Farad</td>
<td>$F$</td>
<td>Coulomb per Volt</td>
<td>C/V</td>
</tr>
</tbody>
</table>

9.3 Experimental set-up

Download the Capstone setup file from the lab’s website at
http://groups.physics.northwestern.edu/lab/em-oscillations1.html

Insert the components into the breadboard of your setup as shown in Figure 9.6. We will measure a $0.1 \mu F$ capacitor and a $1.0 \mu F$ capacitor, but it is not important which we observe first.

We will also need to install the inductor, the function generator, and two channels of our oscilloscope.

Observe the shape of the input pulses through Channel A of the computer-based oscilloscope display, while simultaneously observing the capacitor’s voltage, $V_C$, with Channel B. Power on Pasco’s 850 Universal (computer) Interface and start their Capstone program. Click the “Record” button at the bottom left. You should see a decaying oscillation quite similar to Figure 9.2. If you do not, check your circuit connections or ask your TA for assistance.

Click the “Signal” button at the left and play with the “Frequency” of the pulse train. You should see that higher frequencies don’t allow the oscillations time to decay completely before
they are re-energized. The waves are sketched in Figure 9.7. Is the beginning point constant or does it jump around? We want to use a frequency low enough that the measurement noise is almost as large as the oscillations before the pulser changes state; this will cause a repeatable waveform that is the same every time.

9.4 Procedure

Sketch the displayed $V_C(t)$ in your Data; some artistic skill will be useful from time to time. Be sure to note which capacitor you are using. What are the tolerances and units? Indicate on this sketch, using small squares as markers, the points where the energy of the system is all in the electric field of the capacitor. Indicate with a circle where it is all in the form of magnetic energy. Draw a legend that defines what these symbols mean. Use the oscilloscope to measure the period of oscillation, $T$. See if you can remember how to use the Measuring Tool. It may be more accurate to measure the time of $n$ periods, rather than just one. Calculate the angular frequency $\omega' = \frac{2\pi}{T}$.

Hover the mouse cursor over the graph so that a toolbar appears at the top of the graph display. Click the Selector Tool (an icon with a blue square surrounded by eight small grey squares) and adjust the selected area so that only one decaying sinusoid is yellow. Scroll the table until you find the beginning or the end of the yellow entries; these are the table entries that correspond to the yellow data points on the graph. Drag with the mouse to select only these yellow entries and ctrl+c to copy these times and capacitor voltages to Windows’ clipboard.

9.4.1 Observe the ‘Motion’

Run Vernier Software’s Graphical Analysis 3.4 (Ga3) program. We have also provided a suitable setup file for Ga3 on the website. Click row 1 under Time and ctrl+v to paste your Pasco data into Ga3. After a few seconds, Ga3 should plot your data points and automatically fit them to

$$V_C \exp(-g*t) \cos(w*t+p)$$
Compare this expression to Equation (9.16). Since Ga3 cannot accept Greek symbols, it was necessary for us to use English letters: $\gamma \leftrightarrow g$, $\omega' \leftrightarrow w$, and $\varphi \leftrightarrow p$. Be sure the solid fit model curve passes through all of your data points; the computer is not smart enough to do this for you. When manually fitting, keep in mind that $V_{ac}$ is the initial amplitude (half of the peak-to-peak voltage), $g$ is the rate that the amplitude decays, $w$ is the oscillation frequency, and $p$ is a horizontal phase offset. If the curve goes too quickly to 0, reduce $g$; in fact you can use $g = 0$ until you get the oscillation adjusted if you like. To get an approximate $w$, just type in a number (1,2,3...) and add zeros until the oscillations get too fast; now that you have the correct order of magnitude you can fine tune from most significant to least significant digit until the frequency matches that of your data. Choose $-\pi < p < \pi$ since this is the period of sine and cosine; align the zeros with your data’s zeros. Finally, adjust $g$ to get the correct amplitude envelop time dependence. Once the fit model matches your data points pretty well, click ‘OK’. Now “Analyze/Curve Fit...” again and “Try Fit” again to let the autofitter try again beginning with your manual fit parameters. Once your fit is satisfactory, type your names, which capacitor was measured, and any error information you have on the data points into the Text Window. “File/Print/Whole Screen” and enter the number of people in your group into ‘Copies’.

Now swap capacitors and repeat the recording, copying, and fitting procedure for the new oscillations. You will probably want to “Data/Clear Data” in Ga3 before pasting in the new data in case the new data set is smaller than the old one. In this case several data points at the bottom will not be replaced by the new data and they will appear erroneously on your graph and analysis.

9.4.2 Predict the Inductance

Now, use these fitting parameters to determine your circuit parameters. Since there are three unknowns, $R$, $L$, and $C$, but only two equations, it is necessary for us to assume a value for one of our circuit elements. If we measure the inductor’s dimensions carefully, our value of $L$ will have the tightest tolerance of 1%. While measuring the solenoid, keep in mind that it is the wire that results in inductance; the plastic coil form adds nothing. It is therefore necessary to determine the dimensions of the wire coil. The wire was wrapped around the plastic spool so the inner diameter of the wire is the outer diameter of the round plastic tube. The flat plastic ends of the spool holds the ends of the wire coil flat so the length or height of the wire solenoid is the same as the distance between the inside surfaces of these flat spool ends. The outer diameter of the wire solenoid can be measured by looking through the transparent plastic spool ends. The number of turns of wire in the coil is written on the coil’s label along with other useful information. Record your measurements, their uncertainties, and their units.

Execute the “Inductance” applet from the website. Type your measurements into their respective edit controls and click “Calculate”. It is necessary that your measurements be entered with millimeter (mm) units, so you might have to convert your units. Additionally, the program reports the inductance with milli-Henry (mH) units instead of Henrys.
9.4.3 Measure $R$ and $L$

Note the manufacturer’s specified capacitance, $C$; both capacitors have 5% tolerance. This specified capacitance, $\gamma = g$, and $\omega' = w$ can now be used to find a predicted value for your inductance and resistance equivalent of total energy losses. Rearrange Equations (9.16) to see that
\[
L = \frac{1}{(\omega'^2 + \gamma^2)C} \approx \frac{1}{\omega'^2 C}
\]
(9.18)
can be used to predict $L$ and
\[
R = 2\gamma L
\]
(9.19)
can be used to predict $R$. Since the fitting parameter uncertainties are so much smaller than 5%, the uncertainties in these measurements is rather close to 5% as well: $\delta L \approx 0.05 L$ and $\delta R \approx 0.05 R$.

Observe both capacitors and measure $R$ and $L$ twice—once for each capacitor. Be careful that you do not mix up your predictions and measurements. Did you remember to record the manufacturer’s measurements, units, and tolerances for these components? What components have substantial resistance that the current must flow through? Do you get the same inductance and resistance for both capacitors? Discuss this with your classmates to see if there is a correlation that might be a clue to the explanation. Try to find all of the missing pieces.

9.4.4 The Infinite Solenoid

If time is short, skip this part. Determining the inductance of a finite length solenoidal coil is quite difficult, but we have used Ampere’s law to determine the inductance of an infinitely long solenoidal coil to be
\[
L = \frac{\mu_0 AN^2}{h}.
\]
(9.20)
Pretend that this applies to your coil and substitute your dimensions into this formula to predict $L$. We won’t expect to get the correct answer, but we should get within a factor of 2 or 3 if the infinite solenoid formula is correct.

9.5 Analysis

Consider your measured $R$’s and $L$’s; did you get the same answers for both capacitors? Use the strategy in Section 2.9.1 to decide whether each prediction and specification are equal. Do you see a pattern?

What other subtle sources of error have we omitted from our comparisons? Might some of these be large enough to help explain our disagreement(s)? Since $L$ and $C$ store energy and return it to the circuit later, only $R$ dissipates energy. Every loss will increase $R$; can you think of any ways energy was lost from the circuit? Does your large or small capacitor
circuit have larger losses? This might be a clue you can use to locate some of the losses. Repeat this analysis for the inductor keeping in mind that the units are now H instead of \( \Omega \).

Discuss how well our solution fits our data? Is it likely that our fit would be this good if our theory were substantially incorrect? Consider the complexity of our model when answering this. Did we get the right answers for \( L \)? How about \( R \)? Is the same energy initially stored for both capacitors?

9.6 Conclusions

What equations have we demonstrated? References to equations in your main report are preferred. Communicate using complete sentences and define all symbols. Have we made any measurements that we might need in the future? If so note them, their uncertainties, and their units. What is the significance of the initial voltage and phase? How might our readers obtain them from their own data? What does this experiment indicate about the way we analyze circuits? What does our data say about the inductance of an infinitely long solenoid? How might we improve the experiment? Can you think of any applications for what you have observed?