Chapter 6

Experiment 4: Electric Currents and Circuits

6.1 Introduction

The resistance to the flow of an electric current is essential in the design of electronic devices and electric circuits generally. We study in this lab session how the electric current distributes itself when alternative competing paths for the electric current offer different resistances to the flow of charge.

6.1.1 The Electric Current

In a previous lab we learned that the current passing through a wire is defined by how much charge passes through a cross-sectional area, \( A \), of the wire per unit time. By convention, the direction of the electric “conventional current” is taken to correspond to the flow of positive charges, in the direction of the applied electric field from the higher to the lower electrical potential, even when the actual charge carriers are negatively charged electrons, which in reality must move in the opposite direction.

In a previous lab we learned about Ohm’s law. Ohm defined the resistance, \( R \), of a conductor as the proportionality constant in this law, given by the voltage divided by the current

\[ V = IR. \]  

(6.1)

Anything that has electrical resistance may be termed a resistor. In a circuit diagram, the schematic symbol for a resistor is a zigzag line (see Figure 5.4).

The resistors used in most applications are called “carbon composition” resistors - small cylinders with wire leads at each end, constructed of powdered graphite mixed in clay and compressed into molds. Most resistors are color-coded with a series of bands on them, starting from closest to the edge as shown in Figure 5.4 and Table 5.1. The last band, which is gold, silver, or simply absent, denotes the manufacturer’s tolerance in achieving the indicated value of resistance. The band just before that indicates a power of ten multiplying the previously coded numbers. The remaining colors, starting from the edge, indicate a
sequence of digits. An example of this coding scheme is shown and explained in Figure 5.4.

To compute the power expended in moving a current flowing in a resistor, we note that the work the electric field does on a charge $dq$ that “falls” through a potential difference $V$ is $V \cdot dq$. Hence, the work done per second is

$$P = V \frac{dq}{dt} = VI = I^2R = \frac{V^2}{R}$$

and is in units of Volt-Amperes or Watts. The units of Volts (V) and Amperes (A) have been defined in a way that relates electrical and mechanical quantities, because a Volt is an amount of work per Coulomb (C), so that

$$1 \text{ Volt-Coulomb} = 1 \text{ Joule} = 1 \text{ Watt-second} = 1 \text{ Newton-meter}$$

The conservation of electric charge lead the German physicist Gustav Kirchoff to note that every circuit node must have

$$0 = \sum_{i=1}^{N} I_i$$

Figure 6.1: Examples of series and parallel circuits containing three resistors and source of emf.

The fact that the electric force is conservative led Kirchoff to realize that every closed loop must have

$$0 = \sum_{i=1}^{N} V_i$$

where element $i$ causes an increase (+) or a decrease (-), $V_i$, in electric potential. This is because the work done by conservative forces depends only upon the endpoints of the path
traveled. If the two endpoints are the same point, as is necessary for completing a loop, then the work done must vanish. If we take any charge, \( q \), and carry it around the closed loop under discussion, the work is described by multiplying Equation (6.3) on both sides by \( q \).

These relations are called Kirchoff’s node (or current) rule and Kirchoff’s loop (or voltage) rule, respectively. They are irreplaceable in the analysis of circuits as the following discussion will indicate.

The total resistance, \( R_T \), of a complex combination of interconnected resistances is obtained by dividing the voltage applied across the combination by the total current flowing through the combination. Two special cases are the series circuit and parallel circuit (Figure 6.1). In each of the resistance combinations shown in Figure 6.1, the total resistance is \( R_T = \frac{V_B}{I_B} \) where the subscript refers to the element (‘battery’ in this case). For the series circuit, the same current, \( I = I_B \), flows through each resistor and the battery, but the potential difference across the entire series \( V_B = I_B R_T \) is equal to the sum of individual voltage changes \( V_i = I_B R_i \) across each resistor. This equality directly implies that for resistors in series

\[
R_T = \frac{V_B}{I_B} = \frac{1}{I_B} \sum_{i=1}^{N} V_i = \frac{1}{I_B} \sum_{i=1}^{N} I_B R_i = \sum_{i=1}^{N} R_i
\]  

(6.4)

where the battery increases the potential, the resistors decrease the potential, and we have used Kirchoff’s loop rule around the loop of battery and series resistors.

For the parallel circuit in Figure 6.1, it is the voltage across each resistor that is the same, but now the current, \( I_B \), is distributed among different paths, so that the total current \( I_B = \frac{V_B}{R_T} \) is the sum of currents \( I_i = \frac{V_B}{R_i} \) through each branch. This leads immediately to

\[
\frac{1}{R_T} = \frac{I_B}{V_B} = \frac{1}{V_B} \sum_{i=1}^{N} I_i = \frac{1}{V_B} \sum_{i=1}^{N} \frac{V_B}{R_i} = \sum_{i=1}^{N} \frac{1}{R_i}
\]  

(6.5)

where the battery current enters the positive node, the resistor currents exit the node, and we have applied Kirchoff’s node rule to the positive node.

For many complicated circuits, the resistance can be found by breaking the circuit down into combinations of resistances all in series and resistances all in parallel, using Equation (6.4) and Equation (6.5) to calculate the resistance of each branch and then of each required combination of branches, as illustrated at the end of the example given below. Sometimes also, as in this lab session, the voltages between different points in the circuit can be measured, and Equation (6.4) and Equation (6.5) together with Ohm’s law, Equation (6.1), can be applied to different branches of the circuit to find the current in each.

**Checkpoint**

How do the values of resistors connected in series combine? How do the values of resistances connected in parallel combine?
6.1.2 Example of Circuits with Resistors

The following example illustrates how to calculate the current through different parts of a complicated circuit. A 6 V battery is connected to the circuit shown in Figure 6.2. The known resistances are $R_1 = 100 \, \Omega$ and $R_3 = 1 \, k\Omega$. You are given a voltmeter and must determine the currents $i_1$, $i_2$, and $i_3$ as well as the unknown resistance $R_2$. You could measure the voltage $V_1$ across $R_1$ and you could use Ohm’s law to calculate $i_1$. Assume you measured a value of $V_1 = 2 \, V$ (i.e., 2 Volts). Then

$$i_1 = \frac{V_1}{R_1} = \frac{2 \, V}{100 \, \Omega} = 20 \, mA.$$

The total voltage ($V_B$) across the battery is equal to the sum of the voltages $V_1$ across $R_1$ and the voltage (which we call $V_{23}$) across $R_2$ and $R_3$. Then

$$V_B = V_1 + V_{23}$$

and

$$V_{23} = V_B - V_1 = 6 \, V - 2 \, V = 4 \, V.$$

(You could have measured this voltage with the voltmeter.) Ohm’s law now allows us to calculate $i_3$ because we have just calculated the voltage across $R_3$ and because the resistance is known,

$$i_3 = \frac{V_{23}}{R_3} = \frac{4 \, V}{1 \, k\Omega} = 4 \, mA.$$

To determine $i_2$ we use the principle of conservation of charge, which implies

$$i_2 = i_1 - i_3 = 20 \, mA - 4 \, mA = 16 \, mA.$$

Finally, to determine $R_2$ we use Ohm’s law once more

$$R_2 = \frac{V_{23}}{i_2} = \frac{4 \, V}{16 \, mA} = 250 \, \Omega.$$
As a check, we can calculate the total current the battery must supply. First calculate the resistance $R_{23}$ of the branch containing $R_2$ and $R_3$

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} \text{ or } R_{23} = \frac{R_2 R_3}{R_2 + R_3}.$$  

Then calculate the overall resistance $R_T$ of the circuit as

$$R_T = R_1 + R_{23} = 100\,\Omega + \frac{(250\,\Omega)(1000\,\Omega)}{250\,\Omega + 1000\,\Omega} = 300\,\Omega$$

and find the current using

$$I_B = \frac{V_B}{R_T} = \frac{6\,V}{300\,\Omega} = 20\,\text{mA.}$$

This is the current previously found to be flowing first through $R_1$ and then branching to either $R_2$ or $R_3$.

The power dissipated in a resistor is given by $P = I^2 R$, so that the power dissipated in $R_1$ is

$$P_1 = I_1^2 R_1 = (0.02\,\text{A})^2 (100\,\Omega) = 0.04\,\text{W}.$$  

In the second resistor, the power dissipated is

$$P_2 = I_2^2 R_2 = (0.016\,\text{A})^2 (250\,\Omega) = 0.064\,\text{W},$$

and in the third resistor,

$$P_3 = I_3^2 R_3 = (0.004\,\text{A})^2 (1000\,\Omega) = 0.016\,\text{W}.$$  

The total power expended maintaining the current is then

$$P = P_1 + P_2 + P_3 = 0.12\,\text{W}.$$  

To check the calculated results, we can instead evaluate the total power directly from the total current $i_B = 0.02\,\text{A}$ through the battery and the potential difference $V_B = 6\,\text{V}$ across its terminals

$$P = V_B I_B = (6\,\text{V})(0.02\,\text{A}) = 0.12\,\text{W}.$$  

This agrees with the previous value obtained.

**Checkpoint**

By what factor does the power increase when the current through a resistance is doubled? Which branch of a parallel circuit consisting of two unequal resistors in parallel gets the greater current?
CHAPTER 6: EXPERIMENT 4

Checkpoint

In a series circuit containing two unequal resistances which resistance has the greater voltage drop across it?

6.2 Apparatus

The plug-in breadboards we will use in this experiment allow you to assemble your own resistance network to investigate the electronic properties and principles of parallel (Figure 6.1(a)) and series (Figure 6.1(b)) resistance circuits. In this lab we will assemble a circuit of each type using one each of resistances with values of 1kΩ, 470Ω, and 100Ω. To prepare for the lab you should calculate the voltage across each of the series resistors and currents through each of the parallel resistors for a supply voltage of 6 Volts. The resistances you will use are rated for 2.0 Watts. Check to make sure this power rating is not exceeded for any arrangement of resistances you will be using BEFORE you power up the circuit for real.

6.2.1 The Parallel Circuit

Assemble the circuit on the plug-in board using resistances of 1kΩ, 470Ω, and 100Ω in parallel and shorting jumpers (jumpers are plug-in connecting wires with negligible resistance used to interconnect circuit elements) as shown in Figure 6.3. Each of the five sockets in the array connected by a cross is internally connected with conductors. Circuit elements can be interconnected by bridging across the gaps between each of these arrays of sockets. The Pasco 850 Hardware Interface contains a variable power source that we will set to 6.00V. The jumpers to the right of each resistor may seem unnecessary but will have a function. We will be measuring the current through each of the branches of the circuit as well as the voltage drop across each resistance element. The voltages and currents will be measured using 850 Interface sensors. The 850 Interface will report its readings to the computer via the USB protocol and the computer will display the results on the video monitor.

WARNING

You should not attempt to measure the current directly from the power source. Do not connect the leads of the ammeter across the power supply terminals or any leads coming from the power supply. Doing so will blow the inline fuse and cause the ammeter to malfunction. It is also possible that the fuse will not protect the meter and that the current sensor will be destroyed entirely. If you have any doubts about your connection, have your Lab Instructor look over your connections before you turn on the power.

Click the ‘Record’ icon at bottom left to begin gathering data. Now place one voltmeter lead in each of the two places you want to know the potential difference between. Begin by
placing the leads across the power supply and recording the ‘Total’ voltage $V_T$. Record the reading in your data table exactly as the meter displays it. What is your experimental error and units? Be sure to note these as well. \textit{Once this is recorded do not change it again} or all of your voltages and currents will also change to the new value instead of the one you recorded. Now measure and record all three resistor voltages by placing one meter lead on each side of the relevant resistor.

To measure the current in a branch, the circuit must be broken and an Ammeter inserted as shown in Figure 6.3. The ammeter cannot know what the current is unless all of the current passes through the meter. In Figure 6.3 the jumper in the net circuit resistance is removed and the leads to the ammeter are inserted as shown to measure the current through the entire circuit. Simply ignore any negative sign; reversing the leads will eliminate it but the absolute value will be the same in either orientation. Once the current provided by the ‘battery’ is measured, replace the shunt and measure the current in turn through the three resistors in the same way.

Compare the total current with the sum of the three branch currents. If this sum is significantly different from the battery current, you are probably measuring the currents

\textbf{Figure 6.3: Sketch of our apparatus used to construct a parallel circuit. The vertical shunts will be removed and replaced with an ammeter so that the resistor currents can be measured.}
CHAPTER 6: EXPERIMENT 4

Figure 6.4: Sketch of our apparatus used to study a series circuit.

Wrong. Calculate measured values of resistance for all three resistors and for the total circuit resistance using

$$R_i = \frac{V_i}{I_i} \quad (6.6)$$

where $i = 1, 2, 3,$ and $T$. Use the manufacturer’s specified resistances and Equation (6.5) to predict the total circuit resistance. Use your measured resistor currents to predict your battery current using Kirchoff’s node rule,

$$I_T = I_1 + I_2 + I_3. \quad (6.7)$$

Helpful Tip

It would be a good idea to start a “Results” table to hold these calculated predictions. You can add the results from the other two circuits to this table as they become available.
Figure 6.5: Sketch of our apparatus used to construct a series parallel combination circuit. The voltmeter is set to measure the voltage across the parallel lamps. The ammeter measures the battery current.

6.2.2 The Series Circuit

Modify your parallel circuit so that the resistances form a series circuit as shown in Figure 6.4. Note the placement of jumpers and power supply connections. Does the circuit match the series circuit in Figure 6.1(b)? Use the voltage sensor to record the voltages across each of the resistors in the circuit. In Figure 6.4 we show the connections to measure the voltage across the first resistor. Measure the voltage across the battery. Record these voltages, their errors, and their units in your series circuit Data table. Add the three resistor voltages and compare the sum to the battery voltage. If these are significantly different, you are probably measuring the voltages wrong.

Use the current sensor to measure the current being drawn from the battery; Figure 6.4 shows how to do this. Remove the meter leads and replace the meter with a shunt. Remove
one of the shunts and place the ammeter leads in its place to connect the meter in series
with the resistor. Record the resistor’s current and replace the shunt. Repeat this to record
the other two resistor currents. Note the experimental errors and units in your Data table
also.

Use the battery voltage and the battery current to compute a measured total circuit resis-
tance. Use the resistor values measured in “The Parallel Circuit” section and Equation (6.4)
to predict the total circuit resistance. Use your measured resistor voltages and Kirchhoff’s
loop rule to predict the battery voltage,

\[ V_B = V_1 + V_2 + V_3, \]  

and record these predictions in your table of results.

### 6.2.3 The Series-Parallel Circuit

We would like to explore a circuit that uses both series and parallel connections. The circuit
shown in Figure 6.2 is a most useful circuit. For example, the electrical power lines have
resistance, \( R_1 \), and the lights, \( R_2 \), and air conditioner, \( R_3 \), in your home are each powered
by them. When your air conditioner first switches on, your lights dim due to the transient
current needed to start the motors turning. Assemble the circuit using the plug-in board
and jumpers. For each resistance use a socketed incandescent lamp. Ask your lab instructor
to verify the circuit before you continue. We would like to measure the effect of adding the
\( R_2 \) resistance in parallel to the \( R_3 \) resistor, which is connected in series with the \( R_1 \) resistor.
Power up the circuit and note what happens when \( R_2 \) or \( R_3 \) is removed and re-inserted.

BEFORE you come to lab, assume that the resistances of the three identical light
bulbs are equal and compute \( V_1 \) and \( V_2 = V_3 \) if the battery is 6V.

Use the voltage sensor to measure all three lamp voltages and the power supply voltage
and record all four in your series-parallel Data table. Note your experimental errors and
units. Check that \( V_B \approx V_1 + V_2 \approx V_1 + V_3 \); if not you are probably measuring the voltages
wrong. How well do your measurements agree with your pre-lab calculations?

Substitute the ammeter for the shunts one at a time and record the lamp currents.
Remove the meter leads from the circuit and replace all of the shunts. Remove the battery
current shunt and measure the battery current.

Can you think of a real circuit of which this might be representative? There are many!
Here is a partial list:

- Power transmission wires have resistance and power a house with motors and lights
  wired in parallel.
- A voltage divider under load.
- A power supply with internal resistance powering multiple devices.
- Automobile headlights in parallel with starter motor; these are powered by wire and
  battery with internal resistance.
CHAPTER 6: EXPERIMENT 4

Use Ohm’s law and your current and voltage measurements to compute measured values for all three lamp resistances and the total circuit resistance; record this in your Data table. Predict the value of $I_B = I_1$ using $I_B = I_2 + I_3$. Predict the value of $V_B$ using $V_B = V_1 + V_2$ or $V_B = V_1 + V_3$. Predict the value of $R_{23}$ using $\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3}$ and your measured parallel lamp resistances. Predict the value of $R_T$ using this equivalent parallel resistance, the measured series lamp resistance, and $R_T = R_1 + R_{23}$. Record these predictions in your table of results.

Are the three ‘identical’ lamp resistances the same? Can you explain any discrepancies? Are all three very different or are some similar and the other very different? Might this be a clue to understanding the discrepancies? If you think the lamps are different after all, try swapping the different lamp with one of the similar lamps and repeating the measurements of voltage. Are these measurements significantly different than the previous ones? If so, the lamps might indeed be different. Which lamps are brightest now? What might the brightness indicate about the lamps’ resistances.

**Checkpoint**

Which branch of a parallel circuit consisting of two unequal resistors in parallel dissipates more power?

**Checkpoint**

In a series circuit containing two unequal resistances which resistance dissipates more power?

### 6.3 Analysis

Use the strategy in Section 2.9.1 to decide whether your data supports the equivalent resistance formulas and/or Kirchoff’s rules. You will need to do this seven times: 1) parallel current, 2) parallel resistance, 3) series voltage, 4) series resistance, 5) S-P current, 6) S-P voltage, and 7) S-P resistance. Do your predictions agree with your measurements within the 5% resistor tolerances? What other subtle sources of error might have affected your measurements. Might some of these be large enough to explain the remaining disagreement?

Discuss your series-parallel lamp experiment. What supplemental observations have you made (in addition to meter readings) that help explain your data? Refer to your Ohm’s Law report and see if you can prove what is causing the lamps’ resistances to be different.
6.4 Conclusions

Enumerate the four new physical relationships that your data support. (If you have numbered your equations, you may reference relevant ones here instead of re-writing them.) Define all symbols; symbols used in several relations need defining only once. The first circuit had three resistors in parallel \((N = 3)\) and the last circuit had two resistors in parallel \((N = 2)\). The second circuit had three resistors in series \((N = 3)\) and the last circuit had two resistors (one was the parallel combination of two lamps) in series \((N = 2)\). Strictly speaking, we have not tested cases where \(N > 3\), but the same theory was used to model these cases as for \(N < 4\); we might expect, therefore, that the outcomes would be the same when we eventually do check these cases. The first two circuits contained only linear resistances (see Ohm’s law data) and the last circuit contained nonlinear lamps. The theory works in all tested cases. What would you expect for circuits containing LEDs, transistors, and photodiodes that we also haven’t yet tested? The theory works when we have only parallel resistors, only series resistors, and mixed up combinations of series and parallel resistors. We have demonstrated all of this with only three circuits...

What improvements might you suggest for the experiment? Do you see any potential applications for what you have observed?