Chapter 5

Experiment 3:
Geometric Optics

5.1 Introduction

In this and the previous lab the light is viewed as a ray. A ray is a line that has an origin, but does not have an end. Light is an electromagnetic disturbance, and as such is described using Maxwell’s equations, which expresses the relationship between the electric and magnetic fields in an oscillating wave.

Light propagates as a wave, yet many optical phenomena can be explained by describing light in terms of rays, which, in the model for light, travel in straight lines in a homogeneous medium. This model is referred to as Geometric Optics and is a very elementary theory. In this theory, light travels from its origin at a source in a straight line; unless it encounters a boundary to the medium. Beyond this boundary may be another medium which is distinguished by having a speed of light different from the original medium. In addition, light may be reflected at the boundary back into the original medium. A light ray that returns to the original medium is said to be “reflected”. A ray that passes into another medium is said to be “refracted”. In most cases, light encountering a boundary between two media is partly reflected and partly refracted. But the boundary surface can be curved so that various surface elements reflect or refract the light differently. We can arrange these different directions to intersect at a focus and thus to form an image.

Any mirror or lens can be referred to as an optical element. Imaging refers to the process by which a single optical element or a set of optical elements produce an image or reproduction of the light coming from an object. Mathematically, imaging is a one-to-one, onto map between each point on the object with a point in the image. We perceive an object by observing light coming from the object. We are accustomed to the fact that light diverges from an object and travels in a straight line from the object. Our two eyes and our sense of perception use these facts to locate objects in our environment and to create a mental picture of our surroundings.

An optical element can redirect diverging light rays so that they appear to diverge from a different source. Thus an image is formed apart from the original object. The image can
have one of two possible natures. If the light is merely redirected to appear to diverge from a different location, and thus the light, in fact, was not focused at that point, it is said to be a virtual image. The light's source point is not in a virtual image, but the rays' new directions make it seem like they came from the virtual image. The most common example would be the reflection in a plane mirror. The image would appear to be behind the mirror surface, and yet behind the mirror may be a brick wall, hardly conducive to propagation of light. On the other hand, light may be redirected to converge at a point, after which it would diverge from that point in a similar manner to the way it originally diverged from the object. The resulting image is said to be a real image when light actually diverges from the location of the image. A screen or white paper placed at a real image plane will show a duplicate of the object. Perhaps it is obvious that a screen behind a mirror will show no such duplicate.

The principle objective of geometric optics is to be able to determine the location of an image for certain optical elements arranged in a specific geometry. This may be accomplished in two ways. One can sketch key ray paths in a scale drawing of the geometry or one can calculate the image distance and properties using a set of equations. A related activity is choosing optical elements and deciding where to place them to achieve a specific purpose.

5.2 The Experiment

We first use five parallel rays from a ray box to see how lenses and mirrors focus and diverge the rays. We will trace the rays on paper ('ray tracing') and measure the distances between the object and the lens \(d_o\) and between the lens and the image \(d_i\).

5.2.1 The Plane Mirror

The mirror is the simplest of optical elements to understand. Everyone is familiar with the imaging properties of a plane mirror. If you stand in front of a plane mirror you see your image behind the mirror. The location of the image of an object placed in front of a mirror can be diagrammed knowing that the surface of the mirror reflects light with an angle of reflection equal to the incident angle.

Let us look briefly at the properties of the image formed by a plane mirror. It is formed behind the mirror, it is right side up with respect to the object, and it is the same size.

**Figure 5.1:** Light rays reflected by a plane mirror are illustrated. It looks as if the rays originated behind the mirror from a virtual image.
By convention we measure the object distance \( d_o \) as a positive value when in front of the mirror and image distance \( d_i \) when behind the mirror as negative. Hence for the case of a plane mirror where the image is behind the mirror as far as the object is in front then

\[
d_i = -d_o
\]  
(5.1)

The relative size of the image compared to the object is the magnification, \( M \), defined by the relation

\[
M = \frac{h_i}{h_o}
\]  
(5.2)

For a plane mirror the magnification is \( M = +1 \). We emphasize the positive sign because the sign of \( M \) has a significance we will only appreciate later. It tells us if the image is right-side-up (upright) or upside-down (inverted) relative to the object. Figure 5.1 shows a diagram locating the image of a plane mirror. In the case of a plane mirror where the image is oriented the same as the object, \( M \) is positive. Even more interesting is the fact that the magnification is directly related to the relative distances between the object or image and the mirror. In general \( M \) can be calculated from

\[
M = -\frac{d_i}{d_o}
\]  
(5.3)

Using the relation in Equation (5.1) we get \( M = +1 \) again. Such will not always be the case for mirrors in general, but seem to be true for plane mirrors.

We note finally that the rays of light form an observable image because they appear to diverge from a point other than where the object is located. Since our eyes observe diverging rays, we can see the image. For a plane mirror this point is behind the mirror and we can see the image. In fact, the rays cannot actually be traced back to that point; if we try, we get stopped by the mirror. They never really pass through the point of apparent divergence. That point may very well lie inside a solid wall or behind the wall in another room. We refer to the image as a **virtual** image. Not all images are virtual; but all virtual images are erect and have negative image distances in the case of a single mirror.

**Checkpoint**

What does the relative size of object and image depend on?
5.2.2 The Concave Mirror

A mirror surface can be fabricated as part of a spherical surface of radius $R$. By convention a positive radius is one that is measured to the left of the surface assuming light to be approaching from the left to be reflected off the surface. In fact, all distances are positive on the side where light is bouncing around. The reflecting surface would be termed a concave mirror when $R > 0$. The principle axis is a line that strikes the mirror in the center at normal incidence. Any light ray that is parallel to this axis when it is reflected off the mirror will arrive at the same point along the principle axis. This is called the focal point and it can be shown that this point lies a distance $f$ from the mirror where $f = R/2$. These acts provide the circumstances for drawing two rays (1 and 2) diverging from the tip of the object that will re-converge at a specific point to locate the image position for a particular object location. We show how these rays can be used to locate an object in Figure 5.2. A more useful interactive diagram called an applet can be found on the internet at

http://groups.physics.northwestern.edu/vpl/optics/mirrors.html

![Image of concave mirror and light rays](image)

**Figure 5.2:** Light rays reflected from a concave mirror are illustrated. Four rays are easy to draw for mirrors: Incident parallel to the axis passes through the focus after reflection, incident through the focus reflects parallel to the axis, incident through the sphere’s center reflects through the center, hits the mirror on the axis reflects at the same angle. The diverging rays are focused at a real image point.

The applet makes use of a third ray which, if it strikes the mirror normal to the surface at zero incidence angle, is reflected back on itself. A fourth ray can be obtained where the ray strikes the mirror at the vertex, where the principle axis meets the mirror surface. Such a ray is reflected at an angle equal to the angle between the ray and the principle axis.
The most obvious thing about the image formed by this configuration is that the image is inverted. It also appears in front of the mirror. Such an image is so foreign to us that you really have to see it to appreciate it. Take a common metal spoon and hold it at arms length with the concave portion facing you. When you look into the spoon you should see your reflection and everything else behind you in the room inverted. It is harder to discern that the images are in front of the mirror; never-the-less, it is the case. By convention we say that an image located in front of the mirror has a positive image distance. From the definition of magnification in Equation (5.3), \( M \) must be negative if both \( d_o \) and \( d_i \) are positive. This agrees with the fact that the image is inverted.

The image in Figure 5.2 can be located analytically using the expression

\[
\frac{2}{R} = \frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}.
\]  

(5.4)

The image is reduced in size. This further bears out the relation in Equation (5.3) that ties the magnification to the object and image distances. Here the image lies closer to the mirror than the object and hence is smaller. We also note that the rays of light actually pass through the image location and re-diverge. By convention we say that this image is a real image. Visibly it looks like an object is really located at that point, and the phenomenon is the source of many magic tricks and optical illusions. The fact that the image is inverted and located at a positive image distance goes along with the real nature of the image. A real image will be formed if the object is located anywhere between infinity and the focus of the mirror. Real images can be projected. That is, if you were to place a white card or projection screen at the point where the image is formed, the image will appear in good focus on the screen.

If the object is moved to a point closer than the focus of the mirror the rays will no longer re-converge to form a real image. Instead the rays appear to diverge from a point behind the mirror as shown in Figure 5.3. Such an application would be when you use a makeup mirror. Note that the virtual image is enlarged and right-side-up. Virtual images are not projectable, since the rays never come to a focus at any real point in space.

### 5.2.3 Convex Mirror

A convex mirror \((R < 0)\) is often called a divergent mirror. Light rays parallel to the principle axis are reflected away such that they appear to diverge from a focus behind the mirror. An image distance marked off behind the mirror is considered negative and locates a virtual image. In a similar manner according to the convention, a focus behind the mirror is negative, and virtual. Indeed, the rays never do pass through this focus just like rays never pass through a virtual image.

The images formed by a convex mirror are virtual regardless of where the object is placed in front of the mirror. Figure 5.4 shows the ray diagram locating the image. Note that ray 1 appears to diverge from the virtual focus, and ray 2 appears to be headed for the virtual focus though neither ray reaches this focus. Note also that both the radius and the focus
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Figure 5.3: Light rays emitted from a very close-up object are reflected by a concave mirror. In this case the rays do not focus; however, they appear to come from a virtual image behind the mirror. Our eyes can then focus these diverging rays and see the virtual image behind the mirror despite its absence. See the caption of Figure 5.2 to learn how to draw these four easy rays.

would have numerical values with a negative sign.

Checkpoint
What is the difference between a real and a virtual focus?

Checkpoint
What is the difference between a real and a virtual image?

5.2.4 Lenses

Not everyone has personal experience using lenses, or so it might seem. Besides the most common application in eyeglasses used to correct imaging problems, campers, for example, have been know to use a focusing lens to start a campfire without matches using the sun’s energy. It gets more personal than that, however. Each of us uses a natural imaging system
Figure 5.4: Light rays reflected from a convex mirror appear to come from a virtual image behind the mirror.

in our eye to see, which in part relies on a lens within each eye.

The surface of a piece of glass can act as an optical element that changes the direction of a light ray as it passes through the surface. We explored Snell’s Law that describes this phenomenon in the last lab. In most applications of an air-glass surface the piece of glass is thin and the rays quickly encounter the opposite side of the glass at a glass-air boundary where the light emerges from the glass. There are many applications where only this one interface is considered such as looking at objects underwater. The cornea of the eye is an example of such a surface. We will not deal with this imaging process, but consider only applications where the light emerges from the optical element into air again. Such problems are referred to as thin lens applications.

A plane piece of glass with parallel surfaces usually results in a slight sideways displacement of the original beam but no noticeable angular deflection. A pane of glass is transparent and transmits rays from an object undistorted. Such a simple case is worth a quick look compared to its reflection analog the case of a plane mirror. When looking at an object through a pane of glass we are in fact looking at the image of the object which is located at nearly exactly the same spot as the object itself. We have two choices. We can rewrite the imaging equations (5.1), (5.2), (5.3) and (5.4) to include the fact that the image is now on the same side of the optical element as the object, or we can change the convention to say that a virtual image is formed on the opposite side of the optical element from the observer. The accepted convention is to choose the latter. The accepted convention is that a real object (+d_o) is on the side where light enters the lens, but a real image (+d_i), a real radius (+R),
and a real focus (+f) are on the side where light exits the lens. We will see the ramifications of this in the images formed by a thin lens. We should also mention that lenses have two foci; each is a distance |f| from the lens’ center where one is on each side of the lens and both are on the optical axis. Convergent lenses have positive foci and divergent lenses have negative foci.

5.2.5 Convergent Lens

A convergent lens redirects parallel rays such that they come to a focus. It is the lens analog to the concave mirror. An example of a convergent lens is one with two convex surfaces although other combinations of surface curvatures can also form a convergent lens.

![Diagram of a convergent lens showing light rays passing through it, focusing at a point behind the lens.](image)

**Figure 5.5:** Light rays passing through a convergent lens focus at a point behind the lens. Lenses have three rays that are easy to draw: Rays parallel to the axis are focused through the second focal point, rays passing through the first focal point are transmitted parallel to the optical axis, and rays passing through the lens’ center are undeflected.

As we did for mirrors, we define the principle axis to be a line that strikes the lens in the center at normal incidence. Any light ray that is parallel to this axis before it passes through the lens will be redirected to pass through a single point. This is called the focal point and it can be shown that this point lies a distance f from the lens’ center. Unlike the mirror the radius of the surface of the lens is not easily related to the focus; never-the-less, all rays diverging from a point on the object will re-converge at another specific image point after passing through the lens. One of these rays (1) is emitted parallel to the principle axis and is bent to pass through the positive focus on the far side of the lens.

Because a thin lens has two operational sides, (it can be turned around and work just as well), it has a second focus on the same side as the object. Another ray passing through this focus (2) is redirected to be parallel to the optical axis.

We show how these rays can be used to locate an object in Figure 5.5. A more useful interactive diagram called an applet can be found on the internet at
The applet makes use of a third ray that strikes the lens at the vertex, where the principle axis meets the lens surface. Such a ray (3) passes through the lens undeflected.

The most obvious thing about the image formed by this configuration is that the image is inverted just as with the concave mirror. Contrary to the mirror analog, the image appears behind the lens. Such an image is so foreign to us that we really have to see it to appreciate it. Such a convergent lens is not as readily available as a mirror. If you have access to a magnifying glass you can demonstrate the real image formed by such a lens. By convention we say that an image located after the lens has a positive image distance.

Checkpoint

What is a lens?

General Information

Note the simple change in convention from mirrors. From the definition of magnification in Equation (5.3), $M$ must be negative if both $d_o$ and $d_i$ are positive. This agrees with the fact that the image is inverted.

Checkpoint

What is the principle axis of a mirror and of a lens?

The image in Figure 5.6 can be located analytically using the same expression we used for mirrors,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}, \quad (5.5)$$

but the focal length is more difficult to compute for a lens.

The image is reduced in size. This further bears out the relation in Equation (5.3) that ties the magnification to the object and image distances. Here the image lies closer to the lens than the object and hence is smaller. We also note that the rays of light actually pass through the image location and re-diverge. By convention we say that this image is a real image. Visibly it looks like an object is really located at that point, and a plane card placed at that point will display the image projected on it. Real images are projectable for lenses as well as for mirrors. This phenomenon is applied to camera imaging as well as to how our eyes work. The fact that the image is inverted and located at a positive image distance goes along with the real nature of the image. A real image will be formed from a convergent lens if the object is located anywhere between infinity and the focus of the lens.
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Figure 5.6: Light rays emitted from a very close object do not focus; however, they appear to come from a virtual image instead of the object. The ray directed away from the first focus is transmitted parallel to the axis, the ray incident parallel to the axis is transmitted through the second focus, and rays passing through the lens’ center are undeflected. For $d_o < f$, these rays diverge less than before.

If the object is moved to a point closer than the focus of the lens, the rays will no longer re-converge to form a real image. Instead the rays appear to diverge from a point in front of the lens as shown in Figure 5.6. Such an application would be when you use a magnifying glass. Note that the virtual image is enlarged and right-side-up. Virtual images are not projectable, since the rays never come to a focus at a real point in space, but the resulting diverging rays can be focused by our eyes.

5.2.6 Divergent lenses

A divergent lens is the analog of a convex mirror. Light rays parallel to the principle axis get redirected away from the optical axis such that they appear to have originated from the same point. An image distance marked off in front of the lens is considered negative and locates a virtual image. In a similar manner according to the convention, a focus in front of the lens is negative, and virtual. Indeed, the rays never do pass through this focus just like rays never pass through a virtual image. This focus is not to be confused with the real focus of a convergent lens.

The images formed by a divergent lens are virtual regardless of where the object is placed in front to the lens. Figure 5.7 shows the ray diagram locating the image. Note that ray 1 appears to diverge from the first virtual focus, and ray 2 appears to be headed for the second focus though neither ray reaches its focus. Note also that the virtual focal distances...
Figure 5.7: Light rays passing through a divergent lens appear to come from a virtual image instead of the object itself. Three easy rays for a divergent lens are parallel rays are deflected away from the axis and appear to come from the first focus, rays heading toward the second focus are deflected parallel to the optical axis, and rays passing through the lens’ center are undeflected.

are negative.

Checkpoint

What is the difference in the convention for mirrors and the convention for lenses?

5.2.7 Mirrors

Set up the ray box used in the last experiment but use the 5-slit mask. The ray box may have to be adjusted to make the rays parallel; use at least two meters for this adjustment. The piece of the box holding the slits will slide in the piece holding the light and allow you to cause the rays to emerge parallel to each other. Always be sure the rays are parallel while you are measuring the focus of optical elements. Locate the mirror block. It is roughly a triangular silver plastic piece. One side is flat, one side is concave, and one side is convex.

With the flat side placed in the beam of parallel rays observe how the rays are redirected while maintaining the parallel structure. Note too that the deflection angle is twice the incident angle. A small turn of the block results in twice as much change in the direction of the rays. Briefly note your observations in your Data.

With the concave side placed in the beam, observe how the rays are redirected to a focal point and from there re-diverge. It might help to elevate the front or the back of the ray box by placing the ruler and/or the protractor under it. With the convex mirror facing the beam the rays diverge and appear to emanate from a focus behind the surface.
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Place the concave mirror so that the center ray hits the center of the mirror and rotate the mirror until the focused rays are on the optical axis; they intersect the center ray at the same point. Carefully mark the mirror surface with a pen or pencil on the record paper. If the mark is not exactly below the mirror, move the mirror exactly to the mark and verify that the rays still converge on the optical axis. Mark the point where the rays converge and circle the area where one might reasonably choose an alternate focus. It might help to block the center ray at the ray box temporarily. Remove the mirror and paper, turn on your lamp, and carefully measure the mirror’s focal length. The radius of the circle around the focus is a reasonable estimate of the error in this measurement. Record your focal point measurement, units, and error in your Data.

Place the convex mirror so that the center ray hits the center of the mirror and is retroreflected back into the center slit. Carefully mark the mirror surface with a pen or pencil on the record paper. If the mark is not exactly below the mirror, move the mirror exactly to the mark and verify that the center ray still reflects along the optical axis. Mark two points on each incident ray and each reflected ray; separate each pair of points by as much distance as possible to make reproducing the rays’ angles more reliable. Remove the mirror and paper, turn on your lamp, and carefully reconstruct the five incident and the five reflected rays. Extend all rays to the mirror’s surface. Each incident ray should intersect its reflected ray at the mirror’s surface; if this is not the case, use your records on the paper to reconstruct the experiment with the ray box. This will allow you to adjust your record points to agree better with the rays so that they intersect at the mirror as they should. Extend all five reflected rays through the mirror’s surface and a few cm further than they converge at a virtual focus. Mark the point where the rays appear to originate and circle the area that one might reasonably choose as an alternate focus. Use the ruler to measure the mirror’s (negative) focal length. The radius of the circle around the focus is a reasonable estimate of the error in this measurement. Record your focal point measurement, units, and error in your Data.

5.2.8 Lenses

Set up the ray box used last week but use the 5-slit mask. The ray box may have to be adjusted to make the rays parallel; use at least two meters for this adjustment. The piece of the box holding the slits will slide in the piece holding the light and allow you to cause the rays to emerge parallel to each other. Always be sure the rays are parallel while you are measuring the focus of optical elements. Locate the two-dimensional convergent lens (convex surfaces). Place this lens in the beam of parallel rays and observe how the rays converge to a focus on the side of the lens away from the source. Move the lens around and note how the emerging rays respond to the various lens positions. Place the lens perpendicular to the rays such that the center ray hits the lens center. This placement should cause the rays to converge on the optical axis (where the center ray would be without the lens). If this is not the case, adjust the lens position to make this true. Mark both lens surfaces (or the center of both sides) so that the lens center can be located later. Mark the point where the rays converge and circle the area one might reasonably choose an alternate focus.
Remove the lens and paper and turn on your lamp. Use your ruler to reconstruct the center ray; extend this ray back past the lens location. Use your ruler to locate the center of the lens. Carefully measure the distance between the lens center and the focus. The radius of the circle of alternate foci is a reasonable estimate for the error in this measurement. Record your measured focus, its error, and its units in your Data: $f_{c,l} = (f \pm \delta f)$ U.

Locate the two-dimensional divergent lens (concave surfaces).

**WARNING**

Be careful not to drop this lens because its thin center makes it easy to break.

Be sure the rays from the ray box are parallel. Place the lens in the beam of parallel rays and observe how the rays appear to diverge from a point in front of the lens. Move the lens around and note how the diffracted rays respond to the various lens positions. Place the lens perpendicular to the rays such that the center ray hits the center of the lens. When this is true, the center ray should remain on the optical axis. Mark both surfaces of the lens (or the center of both edges) so that the lens center can be located later. Mark two points on each diffracted ray so that each ray can be reconstructed later. Remove the lens and paper and turn on your lamp. Use your ruler to reconstruct the center ray; extend this ray back past the apparent point of origin for the five rays. Use your ruler to locate the center of the lens. Use your ruler to reconstruct all five rays and extend each ray past the lens and a few cm past the optical axis. The extended rays should converge to a point; if not, use your record paper to reconstruct your experiment and to determine how to adjust your points for better agreement. Mark the point of convergence and draw a circle around the area one might reasonably choose an alternate focus. Carefully measure the (negative) distance between the lens center and the virtual focus. The radius of the circle of alternate foci is a reasonable estimate for the error in this measurement. Record your measured focus, its error, and its units in your Data: $f_{d,l} = (f \pm \delta f)$ U.

### 5.2.9 A Lens System

Place the convergent lens in the beam and form a focused beam. Adjust the lens position to optimize the focus. Use the focus as an “object”. Place the divergent lens in the path of the diverging rays at least 6 cm after the focus and observe that the rays are redirected by the lens so that they appear to be diverging from a different point. See position 1 in Figure 5.8. The lens should be perpendicular to the optical axis and the center ray should hit the center of the lens. The center ray will remain on the optical axis when this is true. The virtual image is where the new rays seem to come from. Mark both divergent lens surfaces (or the center of both edges) so that the lens’ center can be located later. Mark the location of the “object” and draw a circle around the area that might alternately be chosen as the focus (i.e. the “object”). Mark two widely separated points on each diffracted ray after the divergent lens. Remove the lens and paper and turn on your lamp. Reconstruct the center ray and extend it back to the “object”. Use the ruler to locate the lens center. Reconstruct the five
diffracted rays and extend each back a few cm past where they cross the optical axis. All five should intersect the optical axis at the same point. If this is not the case, use your record paper to reconstruct your experiment and to adjust your points for better agreement. Measure the (negative) virtual image distance between the lens center and the virtual image. The radius of the circle of alternate radii is a reasonable estimate of the measurement error. Record your measurement, its error, and its units in your Data. Repeat for your “object” distance and calculate the divergent lens’ focal length using Equation (5.5). Does this focus agree with the focus measured directly? We will check this numerically in our Analysis. Since your object distance and image distance each have error, it should be obvious that your calculated focal length also has error due to both of these sources.

![Diagram](image)

**Figure 5.8:** An illustration of two strategies for measuring the focal length of a divergent lens. In (a) the rays that diverge from the first lens’ focus are spread even further by the concave lens. In (b) the rays that would have converged at one focus are redirected to a different focus.

Now place the divergent lens at position 2 in Figure 5.8, between the convergent lens and the focal point and at least 5 cm from the focal point. Observe that the rays converge to a new focal point a little further away from the lenses. This is a real image. But where is the object? The original focus has disappeared when the lens was placed in the beam. It has become a virtual object! Remove the divergent lens and mark the location of the original focus; draw a circle around the area of alternate possible foci. Place the lens in the beam perpendicular to the optical axis such that the center ray hits the lens center. The center ray should remain on the optical axis when this is true. Mark the image location (the new focus point), the area of possible alternate foci, and the divergent lens location. Locate the lens center and measure the object and image distances; be sure to assign the correct signs. Record these measurements, their errors, and their units in your Data. Again use Equation (5.5) to calculate the focus of the lens and use Equation (5.9) to estimate its
error. Don’t forget to record this measurement, its units, and its error in your Data. How well does it match the previous two measurements?

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<tr>
<th>Checkpoint</th>
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<tbody>
<tr>
<td>What is a virtual object?</td>
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### 5.2.10 Lenses

Measure the focus of a real lens. Using a source far away (several meters) find the point where the image of this distant source is brought to a focus on a screen. With the room lights dimmed use the light coming through the lab door or window as an object. How does this compare with the value printed on the lens housing? Keep in mind that your measurement might not be very accurate.

Place the optical source at one end of the optical bench and place the white screen on the opposite end of the optical bench. Place a convergent lens between the source and the screen but nearer to the source and find the spot where the image of the source projected onto the screen is in focus. Observe that the image is larger than the source because the image distance is larger than the source distance to the lens. How far can you move the lens before the image is noticeably out of focus. Use this distance as an estimate of your measurement error.

Record the lens location, its error, and its units: \( l_1 = (x \pm \delta x) \) U. Calculate the object distance \( d_o \) and the image distance \( d_i \) and use Equation (5.5) to find the focal length of the lens. Is it comparable to the value observed above using the open door?

Now we will measure the focus in another way. There are two possible locations where a focused image can be achieved. Move the lens closer to the screen until a second smaller focused image appears on the screen. What has changed? Note the position of the lens for this geometry. How far can you move the lens before the image is noticeably out of focus? What does this distance mean? Record this lens location: \( l_2 = (x \pm \delta x) \) U. Calculate the object distance \( d_o \) and the image distance \( d_i \) and use Equation (5.5) to find the focal length of the lens.

Calculate the distance, \( d = |l_2 - l_1| \), between the positions of the lens for which the image
was in focus as shown in Figure 5.9. Estimate the error in this distance using the errors in the two lens positions,

$$\delta d = \sqrt{(\delta l_1)^2 + (\delta l_2)^2}. \tag{5.6}$$

This distance can be used as another way to measure the focal length of the lens. Measure the total distance $D$ between the object and the screen. Estimate the error in this distance $\delta D$. The focal length of the lens is calculated using

$$f = \frac{D^2 - d^2}{4D}.$$

5.3 Analysis

We have measured three different values for the focal length of the diverging cylindrical lens. Label them such that $f_1 < f_2 < f_3$ and use the strategy in Section 2.9 to compare the smallest and largest with the middle one.

We have also measured three different values for the focal length of the real lens on the optical bench. Repeat the analysis above for these three values. For each of these four experiments, determine whether the focal lengths are statistically distinct. Statistics can tell us that the two numbers are probably not equal, but statistics cannot tell us why. Perhaps we have underestimated one or more of our errors. Perhaps the devices in our experiments have properties that contribute to our data and of which we have not properly compensated. Perhaps the environment in which we performed our experiment has contributed to our data. Perhaps our hypothesis is, in fact, false. As experimenters, we must evaluate all possibilities and decide which is the truth. Enumerate some of these other sources of error and estimate how each affects your data. In your view is it more likely that we have underestimated how these other effects affected our data or that the theory we tested is false? Keep in mind that others have investigated these theories also and that their results are in the literature.

5.4 Conclusions

What physics do your data support? What do they contradict? What did you test and fail to resolve? Do your data support Equation (5.5)? Might Equation (5.3) be true? We did not test Equation (5.3) numerically, but your observations on the optical bench might allow you to address this qualitatively. Always define the symbols used in these equations; it should not be necessary for your readers to read anything else to understand your Conclusions fully.

Are there any limitations on the focus equation? What can be used as objects? Did you measure anything whose value might be useful later? The focal lengths of lenses and mirrors might be useful information if we should use these optical elements again. Always include measurement errors and units in these values.
5.5 Appendix

We can use Equation 2.9 to calculate the uncertainties in focal length. To make this apply to Equation 5.5, we introduce the intermediate variables \( x = \frac{1}{f} \), \( x_o = \frac{1}{d_o} \), and \( x_i = \frac{1}{d_i} \). Since the numerators are exactly 1, \( \frac{\delta x}{x} = \frac{\delta f}{f} \), \( \frac{\delta x_o}{x_o} = \frac{\delta d_o}{d_o} \), and \( \frac{\delta x_i}{x_i} = \frac{\delta d_i}{d_i} \). Using these definitions Equation 5.5 may be written

\[
x = x_o + x_i
\]  

(5.7)

and

\[
\frac{x\delta f}{f} = \delta x = \sqrt{\delta x_o^2 + \delta x_i^2} = \sqrt{\left(\frac{x_o\delta d_o}{d_o}\right)^2 + \left(\frac{x_i\delta d_i}{d_i}\right)^2}
\]

(5.8)

from Equation 2.8. We use these intermediate relations again to restore the original variables. Calculate the error in your focal length using

\[
\delta f = f^2 \sqrt{\left(\frac{\delta d_o}{d_o^2}\right)^2 + \left(\frac{\delta d_i}{d_i^2}\right)^2}.
\]

(5.9)

The errors in the focus calculated using Equation (5.6) due to the errors in \( d \) and \( D \). Use the errors \( \delta d \) and \( \delta D \)

\[
\delta f = \sqrt{\left(\frac{d}{2D} \delta d\right)^2 + \left(D^2 + \frac{d^2}{4D^2} \delta D\right)^2} \approx \frac{d}{2D} \delta d.
\]

to estimate the error in \( f \). As a challenge, the student might derive this result using Section 2.6.3.