Chapter 7

Experiment 5: RC Circuits

7.1 Introduction

A capacitor, often referred to as a condenser, is a simple electrical device consisting of two nearby conducting surfaces separated by an insulator. The capacitor can store charge of opposite sign on the two plates, and is of immense importance in the design of electronic devices. The purpose of this lab session is to examine how a capacitor stores its electric charge and how it discharges by producing a current through a resistor.

Historical Aside

The ability of a capacitor to store charge was discovered accidentally in two independent experiments at different places during the same year. The first discovery was by Ewald Georg von Kleist in October of 1745 when he tried using an electrostatic generator to place a charge on an iron nail inside a small glass bottle. Later that same year, Anreas Cuneus, a lawyer who frequently visited one of the laboratories at the University of Leiden, was trying to electrify water. He used a chain hanging into a flask of water, and brought the end of the chain into contact with an electrostatic generator. In both cases, after disconnecting the generator, the experimenter touched the metal nail or chain inside the flask with one hand while the other hand still surrounded the outside of the container, and the experimenter experienced an electric shock as a result.

The second discovery, in Leiden, led to the earliest commonly-used capacitor, known thereafter as the “Leyden jar.” It consisted of a metal chain to conduct charge to a sheet of metal on the inside bottom of the jar, with the outside bottom surrounded by a second sheet of metal. The Leyden jar was widely used by early experimental investigators. No longer was it necessary to connect their experimental apparatus directly to the electrostatic generator. Charge could instead be placed in the jar, and carried to the experiment.
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**Historical Aside**

Franklin, for example, used a Leyden jar to collect charge during a thunderstorm in his famous kite experiment to show that lightning was an electrical phenomenon.

![Figure 7.1: Illustration of a modern parallel plate capacitor.](image1)

**Figure 7.1:** Illustration of a modern parallel plate capacitor.

In both experiments the hand holding the container served as a conductor connected to a large reservoir for charge, namely the experimenter’s own body, so his hand around the container acted as one of the plates of the capacitor, his body served as the ground, and the metal inside acted as the second plate separated from the first by the glass insulator. A net charge placed on the inner conductor produced Coulomb forces acting to induce a charge of opposite sign on the hand around the container, with the excess charge free to flow into or out of the ground. Removing the container from the generator left the metal inside and the experimenter’s hand surrounding it outside with opposite net charge. But when the experimenter touched the inner conductor with his other hand, a sudden surge of charge could flow from one conductor to the other though the experimenter’s body, with shocking results.

![Figure 7.2: A schematic diagram illustrating three capacitors in parallel. The positive plates are shorted together so their potentials are equal; the same is true for the negative plates.](image2)

**Figure 7.2:** A schematic diagram illustrating three capacitors in parallel. The positive plates are shorted together so their potentials are equal; the same is true for the negative plates.

**Historical Aside**

The early ideas of the Leyden jar condensing the electrical fluid (or charge) of Franklin’s single fluid theory, and of using jars whose capacities would normally be measured in pints and quarts, is the origin of the commonly used terms “capacitor” and “condenser.”
The capacitors used in modern electronics are less cumbersome than the Leyden jar, though the principle of operation is the same. In a simple parallel plate capacitor (Figure 7.1), two nearby conducting plates are separated by an insulator between them. If the two plates are connected to opposite terminals of a battery, one plate acquires a positive charge, \( Q \), and the other a negative charge, \(-Q\). The electric field between the two plates, and therefore the potential difference between them, is proportional to the charge \( Q \) producing the field.

The capacitance, \( C \), whose value depends on the detailed construction of the individual capacitor, is then defined to be the proportionality constant, \( C = Q/V \), and its value states how much charge is stored per volt of applied potential difference. By convention, we take \( Q \) in this expression to be that on the positively charged plate, so that \( C \) is always positive. The unit of capacitance is the Farad, named after Michael Faraday. The SI symbol for the farad is \( \text{F} \), and one Farad is equal to one Coulomb per Volt. It happens, however, that 1 F is a truly huge capacitance, and typical capacitors are more conveniently measured in microFarads, with \( 1 \mu\text{F} = 10^{-6} \text{F} \), sometimes even in picoFarads, with \( 1 \text{pF} = 10^{-12} \text{F} \). But because the commonly used abbreviation for “micro-Farad” originated before present conventions for the names and abbreviations of physical units were uniformly followed, the commonly encountered abbreviations “MF” and “mf” have also come to be used to mean “micro-Farad,” rather than “milli-Farad” as SI conventions would require and “mmf” or “micro-micro-Farads” was common parlance for pF. Capacitors in older devices will use these markings.

In order to charge the capacitor, a battery or generator must move charge from one plate of the capacitor to the other through a potential difference thereby doing work. Thus a capacitor not only stores opposite charge on its two plates, but also stores electric energy. As we move charge through the circuit to build up the potential difference across the capacitor, the potential difference through which the charge must be moved at each instant is

\[
V(t) = \frac{Q(t)}{C}.
\] (7.1)

For any one specific capacitor, the energy stored in it should depend only on its final charge, not on the history of how the charge was built up. Therefore, to determine the energy stored, we can assume the charge was built up at a constant rate starting from zero and make the analysis easier. Then \( V_{\text{avg}} \), the average value of the potential over this time interval, is half
Figure 7.4: In (a) is the schematic diagram of a capacitor discharging through a resistor. This yields the blue trace in (b). When the switch, $S$, is flipped to position 1, the battery re-charges the capacitor and gives the orange trace in (b).

Capacitors, like resistors, can be connected either in series as shown in Figure 7.3, in parallel as shown in Figure 7.2, or in complex arrangements that can be broken down into combinations of parallel and series sets of capacitors. For capacitors in parallel (Figure 7.2), the voltage $V$ across each capacitor is the same and the total charge stored $Q_T = C_T V$ is the final $V$, and is therefore equal to $V_{avg} = \frac{V}{2} = \frac{Q}{2C}$. We would expect $V_{avg}$ multiplied by the total $Q$ that is passed from one plate to the other through this average potential difference to give the work that was done and the total energy stored, which is then

$$U_C = QV_{avg} = Q \cdot \frac{Q}{2C} = \frac{Q^2}{2C}. \quad (7.2)$$

in agreement with the result previously derived. We have used Equation (7.1) in the last step to substitute for $Q$.

The methods of integral calculus can be used to prove that the work done is indeed the charge multiplied by the time-average of the potential energy, provided the charge and therefore the potential energy difference, are built up at a constant rate.

Calculus can also show that the energy stored in a capacitor is given by Equation (7.2) regardless of the particular function of time, $V(t)$. The work $dW$ done to move a small charge $dq$ through the circuit from one plate to another with potential difference $V(q)$ between them is $dW = V \, dq$. Therefore, the work done to place a total charge $Q$ when voltage $V$ appears across the capacitor is the sum of all the small works on these increments of charge and the potential energy stored is equal to the total work done, or

$$U = \int_0^Q V(q) \, dq = \int_0^Q \frac{q}{C} \, dq = \frac{1}{C} \int_0^Q q \, dq = \left[ \frac{q^2}{2C} \right]_0^Q = \frac{Q^2}{2C} = \frac{1}{2}CV^2. \quad (7.3)$$
the sum of the charges \( Q_i = C_i V \) on the positive plates, then

\[
C_T = \frac{Q_T}{V} = \frac{1}{V} \sum_{i=1}^{N} Q_i \\
= \frac{1}{V} \sum_{i=1}^{N} C_i V = \sum_{i=1}^{N} C_i. \quad (7.4)
\]

Capacitors in parallel add to yield an equivalent larger capacitance like resistors in series add to form a larger equivalent resistance.

For capacitors in series, the charge on opposite plates is \( \pm Q \) and charge conservation dictates the same \( Q \) for each capacitor. The voltage \( V_T = Q/C_T \) across all of the series is the sum of the potential differences \( V_i = Q/C_i \) across each so that in Figure 7.3

\[
\frac{1}{C_T} = \frac{V_T}{Q} = \frac{1}{Q} \sum_{i=1}^{N} V_i = \frac{1}{Q} \sum_{i=1}^{N} \frac{Q}{C_i} = \sum_{i=1}^{N} \frac{1}{C_i}. \quad (7.5)
\]

Capacitors in series have their reciprocals add to the reciprocal of an equivalent capacitance like resistors in parallel. Thus we note an immediate difference between the math for resistors and capacitors.

While the mathematical analysis for capacitors in series and in parallel is strikingly similar to the corresponding analysis of series and parallel resistors, the physical difference between the roles of capacitors and resistors in a circuit should always be kept in mind. The capacitor responds to an applied potential difference by storing charge on its plates that it can later discharge to produce a brief current in the direction opposite to that which charged it like a spring. The capacitor stores energy in the electric field between its plates. Contrarily, a resistor responds to an applied voltage by allowing a current to flow through it and by dissipating energy and converting it to heat like the frictional force. The present experiment studies a combination of both behaviors by examining the short-lived current that a capacitor produces when it discharges through a resistor.

**Checkpoint**

Compare the capacitance of two capacitors to the “equivalent capacitance” of the same two capacitors in series. Which, if either, is bigger? Is there enough information to tell?

**Checkpoint**

When two capacitors are connected in parallel, and the capacitance of one of them is increased, how does the overall capacitance of the circuit change? Why?
Suppose a capacitor in the circuit shown in Figure 7.4(a) has charge $\pm Q$ on its plates. At time $t = 0$ the switch is moved to position 2 and electrons from the negatively charged plate become free to flow through the resistor to the positively charged plate. Charge conservation demands that the rate of charge leaving the capacitor to flow through the resistor is the charge per unit time, or the electric current $I(t) = \frac{dQ(t)}{dt}$, passing through the wire and the resistor. According to Ohm’s law, $I(t)$ is proportional to the voltage, $V_R(t)$, across the resistor at each instant, $t$, and is given by $I(t) = V_R(t)/R$.

When the switch is in position 2, $V_R(t) = V_C(t)$ and is also proportional to the charge still on the capacitor through Equation (7.1), $V_C(t) = Q(t)/C$. If we put all of this together, we see that

$$-R \frac{dQ}{dt} = RI(t) = V_R(t) = V_C(t) = \frac{Q(t)}{C} \quad (7.6)$$

and the rate that charge is being lost from the capacitor’s plates is proportional to the instantaneous charge itself. The negative sign at the start is needed because $Q(t)$ is decreasing while the switch is in position 2. We can divide by $-R$ and use Equation (7.1) to substitute for charge to find

$$\frac{dV_C}{dt} = -\frac{V_C(t)}{RC} \quad (7.7)$$

and the capacitor voltage’s rate of change is also proportional to the voltage itself.

**General Information**

In a great many different physical problems, as here, it happens that the rate of change of some physical quantity is proportional to its value at the time. Such a relation holds also, for example, in radioactive decay as well as in population growth of living cultures.

**Checkpoint**

What effect would decreasing the capacitance $C$ have on the time it takes to deplete half the stored charge and why?

**Checkpoint**

When two capacitors are connected in series, and the capacitance of one of them is decreased, how does the overall capacitance change? Why?

This is such a common situation that the mathematicians have solved it in general. In the Appendix we have detailed some of the reasoning that leads to the solution. Any time
we have a derivative of a function proportional to the function itself like
\[
\frac{df(x)}{dx} = \frac{f(x)}{\lambda},
\] (7.8)
where the constant \( \lambda \) does not depend upon \( x \), the solution to the equation is always the exponential function
\[
f(x) = f_0 e^{x/\lambda},
\] (7.9)
where \( f_0 \) is a constant of integration and can be determined using knowledge of the initial conditions, \( f(0) = f_0 e^0 = f_0 \). In our particular case, \( x = t \), \( f = V_C \), and \( \lambda = -RC = -\tau \). Those familiar with calculus can verify this solution by differentiating Equation (7.9) and substituting into Equation (7.8). Using these results we can immediately write the capacitor’s voltage as an exponential function of time,
\[
V_C(t) = V_0 e^{-t/\tau},
\] (7.10)
with the decay time constant defined by \( \tau = RC \).

**Checkpoint**

Explain, directly in terms of the physical nature of capacitance and resistance rather than from the mathematical form of the formula for the time dependence of \( Q \), \( I \), and \( V_C \) in the \( RC \) circuit, what effect increasing \( R \) would have on how long it takes for half the stored charge to be depleted. Why?

After a very long time, \( t/\tau \) becomes a very large number and \( e^{-t/\tau} = 1/e^{t/\tau} \approx 0 \). After a very long time the capacitor’s voltage and plate charge becomes very close to zero. Suppose we flip the switch back to position 1 while the capacitor is thus discharged. In this case \( V_C(0) = 0 \) and the battery begins to charge the capacitor through the resistor. Kirchhoff’s loop rule yields
\[
0 = V_0 - V_R(t) - V_C(t) = V_0 - RI(t) - V_C(t).
\] (7.11)
From Equation (7.1) we can find that
\[
I = \frac{dQ}{dt} = \frac{1}{C} \frac{dV_C}{dt} = \frac{d}{dt} (CV_C) = C \frac{dV_C}{dt},
\] (7.12)
and Equation (7.11) can be written as
\[
V_0 = RI(t) + V_C(t) = V_C(t) + RC \frac{dV_C(t)}{dt}.
\] (7.13)
In the Appendix we also show that this equation can be solved, that its solution is
\[
V_C(t) = V_0 \left(1 - e^{-t/\tau}\right),
\] (7.14)
and that the time constant is once again $\tau = RC$. We can immediately verify the initial conditions

$$V_C(0) = V_0 \left(1 - e^{-\frac{0}{\tau}}\right) = V_0(1 - 1) = 0 \quad (7.15)$$

are satisfied and we note that these initial conditions are automatically provided by the discharged capacitor resulting from above after a long time. We can also see that after a very long time the capacitor’s voltage is very close to $V_0$. But this is the initial conditions for the development above for a discharging capacitor. Once again we flip the switch back to position 2 and let the capacitor discharge for a long time. In our experiment we will use a square wave generator as the battery and the switch; the square wave generator will flip the switch and wait for a long time, flip the switch back and wait for a long time, flip the switch again and ... until we turn off the function generator.

We will watch the capacitor’s voltage increase and decrease as described by Equation (7.10) and Equation (7.14), respectively, using an oscilloscope and we will measure the time constant, $\tau$, for several combinations of $R$ and $C$. From the manufacturer’s measurements of $R$ and $C$, we will verify that $\tau = RC$. While keeping in mind that $\tau = RC$, we will select one resistor, $R_1$, and we will verify Equation (7.4) and Equation (7.5) with four significant digits of precision. To see how this works we will divide Equation (7.5) by $R_1$,

$$\frac{1}{\tau_s} = \frac{1}{R_1 C} = \frac{1}{R_1} \sum_{i=1}^{N} \frac{1}{C_i} = \sum_{i=1}^{N} \frac{1}{R_1 C_i} = \sum_{i=1}^{N} \frac{1}{\tau_i}. \quad (7.16)$$

Our apparatus can measure $\tau$ to four significant digits of precision if we are careful. We will use $R_1$ for all of our measurements so that only the different $C$’s can be responsible for the different $\tau$’s. If we use different $R$’s, that amounts to dividing the respective different terms in Equation (7.5) by these different $R$’s and we then cannot expect the result to be equal in that case. Algebra demands that we divide every term by the same constant, $R_1$, for the equation to remain valid.

Similarly, we can multiply Equation (7.4) by $R_1$ to find that

$$\tau_p = R_1 C_p = R_1 \sum_{i=1}^{N} C_i = \sum_{i=1}^{N} R_1 C_i = \sum_{i=1}^{N} \tau_i. \quad (7.17)$$

If Equation (7.4) is true and if the rules of algebra are valid, then Equation (7.17) must also be true. Since we can measure $\tau$ to better than four significant digits, we can construct circuits containing individual capacitors and measure their time constants to four significant digits. We can use these capacitors to construct $RC$ circuits with these capacitors in series and then in parallel and we can measure their time constants to four significant digits. We can verify that the time constants obey Equation (7.16) and Equation (7.17) to four significant digits. Since the only difference between Equation (7.5) and Equation (7.16) is the multiplicative constant $R_1$, we have also verified that Equation (7.5) is valid to four significant digits. Since the only difference between Equation (7.4) and Equation (7.17) is the multiplicative constant $R_1$, we have also verified that Equation (7.4) is valid to four significant digits.
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Helpful Tip
This exercise in verifying the equivalence formulas for series and parallel capacitors implements a strategy that is typical in science. It utilizes our ability to measure one quantity (the time constants) well in order to verify a physical relation well. The time constants will be different for every $R$ and the circuit might not emphasize resistance to begin with, but now that we know that the formulas work, they can be applied to any circuit containing capacitors.

Figure 7.5: Sketch of the RC circuit on the breadboard. Be sure to short the grounding tabs together as shown by the red circles. One oscilloscope connector can plug into the side of the function generator connector as shown. This circuit has two capacitors in parallel. Replace one of the shorting jumpers to place the capacitors in series.

7.2 Apparatus
Pasco’s 850 Universal Interface for our Windows based computer can digitize voltage signals and transmit them to the computer. Pasco’s Capstone program accepts these signals and can emulate a computer-based oscilloscope to observe the time dependence of the voltage
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$V_C(t)$ across the plates of the capacitor and the square-wave generator’s output. We have a plug-in circuit breadboard that we can use to construct electrical circuits quickly. Figure 7.5 is a sketch of our circuit showing the square-wave generator that we use for a battery and switch, the series resistor and capacitor, and the oscilloscope needed to observe the square-wave and the capacitor’s voltage. We will adjust the square-wave generator so that for half of its cycle its output is $V_0$ and for half of its cycle its output is 0. Having the square-wave generator output be $V_0$ is exactly the same as having the switch in Figure 7.4 in position 1. Having the square-wave generator output be 0 is exactly the same as having the switch in Figure 7.4 in position 2. We will increase the period of oscillation (decrease the frequency) so that the capacitor has plenty of time to charge and to discharge, respectively, before the switch is flipped to begin the next sequence.

A suitable setup file for Capstone can be downloaded from the lab’s website at

http://groups.physics.northwestern.edu/lab/rc-circuits.html

The computer also is equipped with Vernier Software’s Ga3 (Graphical Analysis 3.4) program. We will copy the data containing the exponentially decaying capacitor voltage to Ga3 and we will allow Ga3 to fit our data to the decaying exponential function. The computer will choose the correct time constant to have the fitted function agree with our data as well as possible. The computer will then tell us what this time constant turns out to be and will also tell us how well the fitted function agrees with our data. If we are careful, we will see that this will allow us to determine the time constants to better than four significant decimal digits. A suitable setup file for Ga3 can also be downloaded from the website.

7.3 Procedure

Begin by connecting the circuit in Figure 7.5 using the plug-in breadboard. The banana jack connectors of the square-wave generator and the Pasco’s voltage sensors have a tab or bump on one side to indicate a connection to the coaxial cable’s shield. The instruments connect this shield to earth ground via the round prong of their power plugs. It is therefore necessary for proper operation that all of these bumps be connected to the shunts at the bottom of the circuit instead of being connected to the resistor. The shunt will connect them all together; since they are all connect to earth ground anyway.

Once the circuit is connected, login to the computer and start Pasco’s Capstone program. Open the lab’s “RC Circuits” webpage and execute the Capstone setup file. Press the
“Record” button at the bottom left to begin the experiment. This causes the 850 Interface to produce a pulse stream that alternates between 0 and 6V.

Helpful Tip

The “Signal” button along the left edge can be used to adjust the pulse rate or the pulse height. Once the new parameters are satisfactory, pressing “Signal” again will remove the control.

The “Record” button becomes the “Stop” button (and vice versa) when it is pressed. Additionally, if you hover the mouse cursor over the graph, a toolbar will appear across the graph’s top that contains a “one shot” icon with a big red ‘1’. Clicking on this button will cause the recording to stop automatically after a single period of output signal. Clicking the button again will toggle the one-shot function off. Once the decaying response is satisfactory, enable the one-shot function to record a single capacitor discharge for further analysis.

You probably now see a square-wave and a decaying exponential. You might also note that they are jumping all over the place. It will probably be necessary to rescale the time axis by dragging the numbers. We need to establish \( t = 0 \) for the waveforms rather than let the computer generate it sort of at random. We do this by triggering the oscilloscope. On the left side of the oscilloscope’s toolbar is a trigger button. Click on this and the graph will then stand still. The trigger symbol will change colors to match Channel A’s data; the selected channel will have a light blue background in the top right of the Scope’s window. You can select the other channel by clicking its square at the top right of the window. Select the channel displaying the square-wave. The de-selected data will be very faint. You might note that the exponential graph is increasing rather than decreasing. We want to study the exponential decay; however, so we need to display the decreasing half of the waveform. Right-click the trigger toolbar button and select “Decreasing” instead of “Increasing”. Now you should see the correct half of the waveforms and the trigger symbol now should point downward.

7.3.1 Observing Exponential Decay on the Oscilloscope

Adjust the waveform to be as much like Figure 7.7(a) as you can manage and note the time constant. You can estimate the time constant by noting that Equation (7.10) predicts that

\[
V_C(\tau) = V_0 e^{-\tau} = V_0 e^{-1} = \frac{V_0}{e} \approx \frac{V_0}{3}.
\]

So, the time constant is the time between the waveform beginning to decay, \( V_C(0) = V_0 \), and the time when the waveform is about 1/3 of the way between the bottom of the square-wave and \( V_0 \). Watch this time interval and try all four combinations of \( RC = R_1C_1, R_2C_1, R_2C_2, \) and \( R_1C_2 \). For each combination record whether the time interval increased or decreased and approximately by what factor. You should try to decide for yourself whether it is possible.
that $\tau = RC$ so record your observations for later review. If $R \to 2R$, does the time constant double? If $C \to C/5$, does the time constant become a fifth as large? If it is not possible that $\tau = RC$, then we obviously do not need to waste time performing a detailed experiment.

7.3.2 Measuring the Time Constant Using an Oscilloscope

Now, we want to pick some combination of $RC$ and to measure the time constant more accurately. It is permissible simply to keep whatever $R$ and $C$ is already installed. First, we need to adjust the function generator’s frequency so that its period is about 5-6 time constants. If the decaying waveform is like Figure 7.7(a), then we want the positive square wave edge just barely off of the display. Click the Measurement Tool button on the graph’s toolbar. Right-click on the Tool’s origin and choose Tool Properties. Increase the Significant
Figures to 5. Click OK. Drag the Tool’s origin to the top left such that the two dotted lines intersect on the exponentially decaying graph. Write the \((t, V_C)\) coordinates in your table as \(t_0\) and \(V_0\). As noted above \(V_C(\tau) = V_0/e \approx 0.3679 V_0\). Multiply the value you measured as \(V_0\) by 0.3679 (or divide by \(e\)) and write the product in your Data table. Now drag the measurement tool until the capacitor voltage is as near as possible to this product. Write this new \((t, V_C)\) in your table and compute your measurement of \(\tau = t - t_0\). If your result is significantly different than \(\tau = RC\), you are doing something wrong. Once your measurement is correct, choose another \((t_1, V_1)\) and repeat the measurement. Are you convinced that every initial point, \((t_i, V_i)\), will yield the same time constant?

As one option we might repeat this process 5-10 times and perform statistics on the time constants (mean and standard deviation). The standard deviation is our best estimate for the uncertainty in each measurement of \(\tau\). And our best estimate of the time constant itself (i.e. the unknown we are trying to measure) is \(\bar{\tau} \pm s_{\bar{\tau}}\) (see Section 2.6.1).

**Checkpoint**

In the experiment, we first determine the time constant by finding the difference between a voltage \(V_0\) and \(V_0/e\). Does it matter which value of \(V_0\) we choose for doing this? Why?

Now use the manufacture’s specified values to predict this time constant

\[
\tau = RC
\]

and estimate the uncertainty in this prediction. Since the only values for \(R\) and \(C\) that we have are known only to within 5%, we expect that the prediction made from these values also has uncertainty that result from these 5% tolerances. We can estimate the uncertainty in our prediction of \(\tau\) from the uncertainties in \(R\) and \(C\) using the product formula, Equation (2.7),

\[
\frac{\delta \tau_{RC}}{RC} = \sqrt{\left(\frac{\delta R}{R}\right)^2 + \left(\frac{\delta C}{C}\right)^2} = \sqrt{0.05^2 + 0.05^2} = 0.071 \quad \text{so} \quad \delta \tau_{RC} = 0.071 RC \tag{7.19}
\]

for the manufacturers’ specified values. Completely specify your prediction for this time constant like \(\tau_{RC} = (RC \pm \delta \tau_{RC}) U\). Be sure not to get your predictions mixed up with your measurements.

### 7.3.3 Measuring the Time Constant Using a Least Squares Fit

Now we want to copy our data into Vernier Software’s Ga3 program for more detailed analysis. At the bottom in the center of Capstone’s area, set the sampling rate to common and about 10,000 samples/s. In the table at left, verify that both time and capacitor voltage have four significant figures. Use the two toolbar buttons above the table to adjust the displayed precision and find the point where the capacitor voltages first decreases to noise.
level (about 0.1 V). If necessary decrease the signal generator frequency to allow \( V_C \) to decay to noise level. Drag the mouse to select all time and capacitor voltage above that point, ctrl-c to copy the data, and paste into Ga3 at row 1 under Time. If data are already in Ga3, “Data/Clear All Data” before pasting to prevent old data from remaining at the bottom of the table.

Ga3 should plot the data points and automatically fit to exponential decay after only a few seconds. Verify that the model line passes through your data points and that the uncertainties are included in the fitting parameters box. Enter your names, the \( R \) and \( C \) used, etc. into the text box, and print a copy for your notebooks. Copy and paste the graph and fit for use in your report. Once you have valid copies, “Data/Clear All Data”, and go back to Capstone to measure the next experiment.

### 7.3.4 Verifying the Capacitance Formulas

**WARNING**

Change *ONLY* the capacitance between fitting experiments from this point forward. DO NOT CHANGE the resistance!

**Helpful Tip**

The next four measurements of time constant are quite precise. Make sure your time constant has uncertainty of a few microseconds and round the uncertainties to two significant digits. Repeat bad data. Make sure you keep exactly the right number of digits in your time constants and that you round them correctly to match your uncertainties. See Section 2.6.4.

Since \( \tau = RC \) and we can measure \( \tau \) very precisely, we basically have a capacitance meter. As long as we do not change the circuit’s resistance, \( \tau \) and \( C \) are always proportional to each other. If we had access to a standard capacitance, we could measure it using our fitting procedure and use this measurement to compute the proportionality constant (\( R \)) needed to make the measurement equal to the standard capacitance. This same \( R \) would continue to relate \( \tau \) to \( C \) until someone changed the resistance; at that point the ‘meter’ would need re-calibrated to the standard capacitance.

The fact that our meter has not been calibrated to yield Farad units does not mean that it is useless. Whatever units (seconds in this case) our meter produces will cancel from Equation 7.4, Equation 7.5, Equation 7.16, and Equation 7.17. Alternatively, measuring \( \tau_1 \) and \( \tau_2 \) in seconds causes Equation 7.16 and Equation 7.17 to predict \( \tau_s \) and \( \tau_p \) in the same units (seconds). We can use our measurements of \( \tau_s \) and \( \tau_p \) then to verify or to contradict those predictions.
Measure the time constants of each capacitor individually, $\tau_1$ and $\tau_2$, and the two capacitors in series, $\tau_M^s$, and the two capacitors in parallel, $\tau_M^p$. Try to get these done quickly so that you can verify Equation 7.16 and Equation 7.17 predicts the $\tau_p^s$ and $\tau_p^p$ that you measured. If the most significant three digits do not agree, repeat the experiment that is responsible for the bad data before continuing. If the parameters box needs more significant digits, right-click the box, “Properties...”, and increase the significant digits as needed. Too many digits can be rounded correctly; too few has already been rounded too much making the computer’s round-off the largest error in the experiment...

Correctly specify all four of your time constants in your Data section: $\tau_1$, $\tau_2$, $\tau_M^s$, and $\tau_M^p$ each should be specified to five or more digits in the parameters box. Round each uncertainty to two significant digits and then round the time constants to match. Make sure they have the same units and round them to the same number of decimal places. Don’t forget to record the correct units as well. If more digits are needed, right-click on the parameters box and increase the number of displayed digits appropriately.

Use Equation (7.17) to predict a value of the time constant for the parallel combination of capacitors,

$$\tau_p^p = \tau_1 + \tau_2,$$

(7.20)

and note that only $\tau_1$ and $\tau_2$ were used to compute this value; finding the uncertainty in this sum is straightforward using Equation (2.6),

Helpful Tip

Be careful to note in your notebook that this result is a prediction. We will compare this with the direct measurement above; but to do so, we will need to know which is which.

Now use Equation (7.16) to predict a value of time constant for the series combination of capacitors,

$$\frac{1}{\tau_p^p} = \frac{1}{\tau_1} + \frac{1}{\tau_2}.$$

(7.21)

Use at least five significant digits of $\tau_1$ and $\tau_2$ and keep at least five significant digits for $\tau_s$. Since $\tau_1$ and $\tau_2$ are used to compute $\tau_s$ and each have experimental uncertainty in them, we expect that $\tau_s$ computed from $\tau_1$ and $\tau_2$ will also have uncertainty in it due to these uncertainties.

### 7.4 Analysis

First, recall that one combination of $\tau = RC$ was first measured manually in Section 7.3.1. This same combination was then fitted to Equation (7.10) and its time constant was measured as a fitting parameter, $(\tau_1 \pm \delta \tau_1)$. Finally, this same combination was then predicted from the manufacturers’ specified component values, $(RC \pm \delta RC)$. We would like to know whether these three measurements of time constant are self-consistent.
Checkpoint

Which of these three measurements yields the best time constant? Why?

Use the best of these three time constants as a standard for comparison and compare
the other two to it. Use the strategy in Section 2.9.1 to decide which of the measured time
constants agree with this standard.

Statistics can tell us that the two numbers are not the same but it cannot tell us why. The
answer to why is ours to figure out. At this level, the student probably made measurement
and/or computational mistakes. It is also possible that our experiment was influenced by its
environment and that we have not adequately allowed for or compensated these influences.
It is possible that our model of our experiment and/or the physical phenomenon under study
have assumptions that are not realized. Finally, it is possible that our theory is wrong; just
look what happened when we tried to use Ohm’s law on an LED!

Discuss subtle experimental errors that have not been included in the $\delta\tau$’s above. If
possible, estimate how each might have affected your measurements. Might some of these
be large enough to explain any observed disagreement?

7.4.1 Notes

When your analysis is complete your notebook should address the following:

- Do your data exponentially decay? How do you know?
- Does our differential equation solution predict the correct behavior and time constant?
- Are Equation (7.5) and Equation (7.4) valid for series and parallel capacitors? How
  well were these equations tested?

7.5 Conclusions

What equations are supported by your data? Define all symbols and communicate with
complete sentences. (If you have numbered your equations in your report, you may reference
them here instead of repeating them.) How well have you ‘proved’ each equation? Have you
measured anything that is likely to be of use in the future? How might you improve the
experiment? How might you apply your observation(s)?
**Helpful Tip**

Review your graphs and your Data section generally to remind yourself what physics we have used in our experiment. If these physics tools led to good agreement, then the tools are supported.

Even if some physics was tested before, we might have used it in a different way or on a different component than before. These new demonstrations are suitable for your conclusions.
7.6 Appendix: The exponential decay law

We use calculus to solve the original differential equation,
\[ \frac{dV_C}{dt} = -\frac{V_C(t)}{\tau}. \] (7.22)

We use algebra to rearrange the equation to the form
\[ \frac{dV_C}{V_C} = -\frac{dt}{\tau}. \] (7.23)

and integrate both sides,
\[ \ln(V_C(t)) = -\frac{t - 0}{\tau}, \]
\[ V_C(t) = V_C(0)e^{-\frac{t}{\tau}} = V_0 e^{-\frac{t}{\tau}}. \] (7.24)

From this we can find the charge on the positive capacitor plate and the current in the circuit.
\[ Q(t) = \frac{V_C(t)}{C} = \frac{V_0}{C}e^{-\frac{t}{\tau}} = Q_0 e^{-\frac{t}{\tau}} \] (7.25)

and
\[ I(t) = \frac{dQ(t)}{dt} = -\frac{Q_0}{\tau}e^{-\frac{t}{\tau}} = -I_0 e^{-\frac{t}{\tau}}. \] (7.26)

When the capacitor is charging, Kirchhoff’s loop rule gives
\[ 0 = V_0 - V_R(t) - V_C(t) = V_0 - RI(t) - V_C(t) = V_0 - RC \frac{dV_C}{dt} - V_C(t) \] (7.27)

or
\[ V_0 = \tau \frac{dV_C}{dt} + V_C(t). \] (7.28)

This inhomogeneous differential equation is harder to solve, but the solution always contains the solution of the homogeneous differential equation that we solved above. The rest of the solution is any particular solution to the inhomogeneous differential equation;
\[ V_C(t) = V_0 \] (7.29)

is such a particular solution as we can see by substituting it into Equation (7.27), so
\[ V_C(t) = V_0 + Ae^{-\frac{t}{\tau}} \] (7.30)
is the solution we seek for some value of $A$. We use the fact that at the moment the switch is flipped the capacitor is completely discharged,

$$0 = V_C(0) = V_0 + A e^0,$$

to find that $A = -V_0$ and our solution is

$$V_C(t) = V_0 - V_0 e^{-\frac{t}{\tau}} = V_0 \left( 1 - e^{-\frac{t}{\tau}} \right). \tag{7.31}$$

From this we can also find the charge on the positive plate and the current flowing in the circuit to be

$$Q(t) = \frac{V_C(t)}{C} = \frac{V_0}{C} \left( 1 - e^{-\frac{t}{\tau}} \right) = Q_0 \left( 1 - e^{-\frac{t}{\tau}} \right) \tag{7.32}$$

and

$$I(t) = \frac{dQ(t)}{dt} = \frac{Q_0}{\tau} e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}. \tag{7.33}$$

We note that the current in the circuit is momentary when charging and when discharging the capacitor and the sign is opposite in the two cases as it should be.