Chapter 8

Experiment 6: Magnetic Force on a Current Carrying Wire

8.1 Introduction

Maricourt (1269) is credited with some of the original work in magnetism. He identified the magnetic force centers of permanent magnets and designated them as North and South poles, referring to the tendency of the North pole to seek the north geographic pole of the earth. A generalization of the interaction between these so-called force centers can be summarized as: opposite poles attract and similar poles repel. Gilbert (1600) recognized that the earth itself was a natural magnet, with magnetic poles located near the geographic poles, only the north geographic pole is a South magnetic pole.

As it was originally conceived, the magnetic field, \( B \), designated the direction and magnitude of the force (per pole strength) on a North pole of a permanent magnet. This was directly analogous to the electric field, \( E \), which was defined as having the direction and magnitude of the electric force (per unit charge) on a positive test charge. The magnetic field emanates from the North pole of a magnet and appears to end on the South pole, but now we believe that \( B \) always forms closed loops.

Ampere determined that a current carrying wire is affected by a magnetic field. Both magnitude and direction of the magnetic force is described by the cross product,

\[
\Delta F_m = I \Delta L \times B,
\]

Figure 8.1: A vector diagram showing the relationship between the current, magnetic field, and resulting force for each piece of the wire.

where the direction of \( \Delta L \) is in the direction of flow of positive current, \( I \). The direction is ascribed to the spatial variable \( \Delta L \) rather than the current, \( I \), for the convenience of integration. If we consider infinitesimal lengths the expression can be written without loss
in generality as
\[ d\mathbf{F}_m = I \, d\mathbf{L} \times \mathbf{B}. \]

Figure 8.1 shows the vector relationship between the magnetic force, \( d\mathbf{F}_m \), the magnetic field, \( \mathbf{B} \), and current direction, \( \mathbf{L} \).

**Checkpoint**

If a current is moving along a wire aligned with a magnetic field, what is the magnitude and direction of the force of magnetism on the wire?

**Figure 8.2:** A sketch of the conductor, the permanent magnets, and the non-uniform magnetic field. For lines through the center, the field is horizontal in our apparatus so that the force can be downward.

In this experiment we will use high strength Neodymium permanent magnets to create an intense magnetic field in a localized region of space. We will place a current carrying conductor in this region, measure the resulting magnetic force, and compare our experimental value with the force predicted by the force equation.

The magnetic field created in the pole gap will be relatively uniform within the gap but will drop off over a finite region just outside the gap. It is necessary to know approximately where the field drops off, to define where along the wire the magnetic field exists, and thus contributes to the magnetic force.

**Figure 8.3:** An example graph of the magnetic field intensity versus the position of the Hall probe.
8.2 Magnetic Field in Magnet Gap

If one had detailed information about the strength of the field as a function of position through the gap, one could do a calculation of the integral

\[ F_m = I \int_0^L dx \times B(x). \]  

(8.3)

Figure 8.2 shows a diagram of the typical magnetic field produced in the gap between two strong Neodymium magnets. A profile of the magnetic field through the gap is shown in Figure 8.3. You will obtain an approximate profile of your specific gap using a Hall probe to measure the field every half centimeter along a line through the center of the gap. For finite segments of distance through the gap we can approximate Equation 8.2 with

\[ \Delta F_i = I \Delta L_i \times B_i \]  

(8.4)

where \( \Delta L_i \) is the unit of length of resolution used to map the magnetic field. A good approximation to the total force can be estimated by summing the components over the range of measured \( B \) field values,

\[ F = \sum_{i=1}^{N} \Delta F_i = \sum_{i=1}^{N} I \Delta L_i \times B_i = I \Delta L \times \sum_{i=1}^{N} B_i, \]  

(8.5)

since the same values of \( \Delta L \) and \( I \) apply to each segment.

**Checkpoint**

What are the units of magnetic force? What are the units of magnetic field?

**Checkpoint**

How is the direction of the magnetic force oriented with respect to the directions of magnetic field and current which produced it?

8.3 Apparatus and Procedure

This experiment will be performed in two parts. First, we will use a Hall effect magnetic field sensor to measure the magnetic field profile, \( B(x) \), between the neodymium (Nd) ceramic magnets. See Section 8.3.1. Second, we will place a current-carrying wire between these same magnets and use a digital mass scale to measure the magnetic force for various charge flow rates (currents). See Section 8.3.3. The apparatus overview is shown in Figure 8.4.
Figure 8.4: A photograph of the apparatus used to measure magnetic field $B(x)$, electric current $I$, and magnetic force $F_m$. Relevant components are indicated.

Hopefully, the Theory above clarifies how these two pieces fit together.

8.3.1 Measuring the B field profile

Two neodymium permanent magnets are placed in the gap of a soft iron yoke to create a very intense magnetic field in the gap between the magnets. The strength and extent of this field will be mapped using a Hall effect probe. First, slide the base of the Hall probe stand until it touches the meter stick guide as shown in Figures 8.5 and 8.6. Keep the base against the meter stick and verify that it passes precisely through the center of the gap between the magnets. If necessary, nudge the magnet yoke as shown in Figure 8.5 to position the center of the gap symmetrically about the probe. Be careful henceforth not to disturb the magnet again until the experiment is complete.

WARNING

The Hall probe has been very carefully adjusted to pass through the center of the magnetic field. It has also been oriented to be most sensitive to the field so do not adjust the Hall probe. Handle the base of the stand gently. If you alter the careful alignment, you can expect to get bad data.
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Slowly and carefully slide the Hall probe through the gap between the magnets to observe the strength of the magnetic field.

**Helpful Tip**

The post holding the magnets and yoke will rotate in the hole in the table. It can be rotated back... When the Hall probe passes cleanly midway between the two magnets while the Hall probe stand is rubbing the meter stick, the magnets are in the right place. Each time the magnets are disturbed, this is the way to put them back. See Figure 8.5.

You should see the field strength increase and then decrease as shown in Figure 8.3. The maximum field should be 550-700 mT if the Hall probe’s alignment is still optimized. Otherwise, tell your lab TA so he(she) can arrange to have it aligned again.

Now we want to measure the profile, $B$ vs $x$, of your magnetic field more carefully. We will use Vernier Software’s Graphical Analysis 3.4 (Ga3) to analyze our data. A suitable setup file can be downloaded from the lab’s website at

http://groups.physics.northwestern.edu/lab/magnetic-force.html

Choose page ‘1:Profile’ for the first part of the experiment.

Carefully slide the Hall probe to the left side of the magnets and find a position where the field strength is about 50 mT. Set the position to be the nearest half-cm to minimize entry errors. Review Figure 8.6 and record the magnetic field and the position of one edge of the probe stand on the meter stick in Ga3. Record other observations in your notebook. Move the probe and stand a half-centimeter along the rule toward the magnets and record the magnetic field and position again. Repeat the operation through the gap until the probe has scanned the entire gap and the field is again about 50 mT. When you finish, your graph should be similar to Figure 8.3. You can also construct a table of $x$ and $B$ in your notebook. Estimate the uncertainties $\delta x$ and $\delta B$ and enter these in Ga3 as indicated also.
Figure 8.6: A photograph of the apparatus configured to measure the magnetic field profile $B(x)$. The peak field strength must be at least 550 mT.

Helpful Tip

Watch for data points that look “out of place”. Your eyes have evolved to notice subtle patterns and departures from patterns, so use the tools you have. Re-measure these bad data points carefully and enter the correct values.

Evaluate the term $\Delta x \sum B_i$ by summing the column of $B$ values in your table and multiplying by the interval, $\Delta x$, you used while making the measurement. Alternately, let Ga3 perform the integral (sum) of the area under the curve to obtain $\text{Area} = A_I = \Delta x \sum B_i$. Draw a box around the data points and “Analyze/Integrate” or click the integrate button on the toolbar.

Checkpoint

What is the Hall probe used to measure in this experiment? If $\Delta x = 1.00 \text{ cm}$ instead of 0.50 cm, why would the calculated area not be twice as big?

Checkpoint

How should we orient the Hall probe to measure the magnetic field most accurately?
(Option)

Use the considerations in the appendix to propagate $\delta x$ and $\delta B$ through to find $\delta A_I$.

### 8.3.2 (Option) The Average Magnetic Field

Calculate the average (or effective) magnetic field for your magnet using your area, $A_I$, and your full width at half maximum, $W$,

$$B_{\text{avg}} = \frac{A_I}{W}.$$  \hspace{1cm} (8.6)

This value characterizes your magnet in case you want to use the magnet in another experiment. We now simply need to measure $W$ and to multiply by this average field instead of measuring all of those fields and integrating. We also need the error in our average field. Can you guess how to calculate this? It will involve $\delta B$ and $\delta x$.

### 8.3.3 Measuring the Magnetic Force

First, carefully remove the Hall effect $B$ field probe and set it aside. (See Figure 8.4.) Next, we need to determine the direction of the magnetic field vector with a magnetic compass. See Figure 8.7. Since the compass magnets can be reversed in the strong field of the Nd magnets, it is first necessary to determine the north pole (N) of the compass magnet. We use earth’s magnetic field to do this as shown in Figure 8.7(a). Hold the compass far from the Nd magnets and thump the handle to randomize any friction. The north pole will point somewhat northward but mostly downward. The blue end in Figure 8.7(a) is the compass’ north pole and will be attracted to the south pole of other magnets. If I were performing the experiment, I would add an entry in my notebook about the color of the compass’ north pole.

#### Historical Aside

The standard convention for magnets is that the north pole is red. Evidently, the magnet in the illustration has had its magnetization reversed at least once.

#### WARNING

The Nd magnets are strong enough to pull the compass’ gimbal apart. Hold the handle firmly and bring the compass close enough to orient the compass’ magnet but not close enough for a strong force. See Figures 8.7(a) and (b).
FIGURE 8.7: Three photographs illustrating how we can determine the direction of a magnetic field. In (a) we use earth’s field to orient the compass. The north pole of the compass (blue) points north and down. (This compass’ magnetization has been reversed.) In (b) the compass’ south pole is attracted to the north pole of the Nd magnet on the left. A little rightward motion makes the magnet on the right closer so that the compass reverses direction. In (c) the compass’ north pole is attracted to the south pole of the Nd magnet on the right. The magnetic field in the gap exits the north pole on the left and enters the south pole on the right as indicated by the blue vectors.

The magnet is attached to a support bar and oriented as shown in Figure 8.4 and in greater detail in Figure 8.7. Now, use the compass to determine which side of the gap has a north pole and which side has a south pole. Hopefully, the two poles are different so that the field does not cancel in the middle. Refer to Figure 8.7 and be careful not to get the compass too close.
Helpful Tip

It is also a good idea to make permanent notes about which end of the compass is attracted to which side of the gap... Such notable facts make great notes. As you figure out which direction $\mathbf{B}$ points, make that observation permanent as well. These details are not needed in the final report; however, they make arriving at a correct report more probable.

An electronic mass scale whose pan has been fitted with a non-ferromagnetic, non-conducting wire support is carefully placed atop a lab jack such that the conductor placed atop the wire support is as shown in Figure 8.4. The conductor and balanced are raised into position such that the wire’s height is the same as the magnetic field’s center; the conductor should now pass through exactly the same points whose magnetic fields we measured earlier—at least as precisely as we can arrange it in the real world. The conductor should also be oriented perpendicular to the magnetic field to maximize the force (recall the vector cross-product).

**WARNING**

The lab jack has been carefully adjusted so that your wire passes through the center of the magnetic field and locked in place. This adjustment is specific to your lab station, so do not move the lab jack to another station. With effort, you can still defeat the jack lock and get bad data... Instead of adjusting the lab jack, refer to Figure 8.8.

Checkpoint

To get the best data, how should we orient the conductor while measuring the magnetic force? What path should the current pass through?

**WARNING**

The mass scale is very sensitive; but it is also very fragile. Any force more than a few Newtons applied to the supports or the weighing pan will break the scale’s sensor.

Slide the lab jack, mass scale, brass wire, and support between you and the yoke. See Figure 8.8. Temporarily lift the brass wire off its supports and lower it between the yoke and the wire supports. Carefully slide the lab jack away from you so that the yoke and magnets are between the supports. Follow the green path in Figure 8.8(a) with the center of the wire as you move the lab jack. It might be necessary to remove objects from between the lab jack and the meter stick. This will allow you to insert the wire between the magnets.
Figure 8.8: Three photographs illustrating how we can insert the current wire and force sensor into the magnetic field. Position the scale as shown in (a), lift the wire, and move the wire toward the magnets and downward so the wire will pass under the magnet yoke as indicated by the green path. Simultaneously, slide the scale toward the magnets taking care that the yoke passes between the two black wire supports (b). Continue sliding the scale until the wire can rest on the supports between the magnets (c).

and to raise it above them so that the supports are on opposite sides of the gap between the magnets. A soft touch is required to avoid moving the magnets and yoke; if necessary, remove the scale and use the Hall effect probe to align the yoke as depicted in Figure 8.5. At this point you can then replace the wire on the supports and have the wire pass through the maximum field between the magnets. Carefully adjust the jack’s position to center the wire between the magnets and align the wire with the front edge of the meter stick. Since the meter stick guided the field probe, this will increase the probability that the current flows along the same path that we profiled earlier. (See Section 8.3.1.)

The mass scale will be used to measure the magnetic force on the conductor as it conducts a current in the magnetic field. These force measurements will have units of grams (g) because we usually want to know the mass of the object placed on the scale and not its
weight \( (w = mg) \). The scale determines these values of mass by measuring the gravitational force (weight) that the mass exerts on the scale. Note that it is the force that the scale senses not the mass. If the scale were in space, it would always read zero regardless of the mass attached to its pan. In this experiment we are tricking the scale by exerting a force that is not a weight; but the scale continues to divide this force by \( g = 9.807 \text{ m/s}^2 \) and to display the result in 'grams'. The scale cannot know that the force we are exerting is a magnetic force; this does not change the fact that to get the magnetic force we need to convert the scale’s 'mass' reading to kg and to reverse the scale’s division by \( g \).

Figure 8.9: A photograph of the current source’s and the mass scale’s displays detailing relevant controls and indications. The current “Hi-Lo” scale switch should be in the “Hi” position and the top current scale should be read. The mass scale photograph shows the decimal point and the gram unit indicator.

Use the right-hand-rule, your observed magnetic field direction, and the direction current flows to predict the direction of the magnetic force. Note that conventional current flows out of the red power supply terminal and into the black terminal. Keep in mind that a previous group might have reversed the leads in the power supply and, if this is the case, the wire colors are backwards.

Set the power supply to 3.0 Amperes and switch it off. See Figure 8.9. Tare the balance to read zero grams. You will need to support the scale while pressing the ‘tare’ button; using one hand to squeeze the button and the scale’s bottom usually works best. Make sure the wires are not in contact with the scale’s sample pan, be careful not to disturb the current wires, switch the power supply back on, and note the effect of 3.0 Amperes on the mass scale. If the balance reading is not positive, your current is flowing in the wrong direction. Verify that the force of magnetism is in the correct direction as predicted by the cross product in Equation (8.1); use the right-hand-rule. Record this vector analysis in your notebook and, perhaps, your Data. To reverse the current, it is far quicker and safer just to reverse the leads.
in the power supply jacks. Read the effective mass as indicated by the scale, and multiply this number by the acceleration of gravity. This is the magnetic force due to the current of 3.0 Amps passing through the conductor in the magnetic field. Take several readings of this force for different currents in 0.5 Amp increments. After setting each current, switch off the power supply and tare the balance for zero current before carefully switching the power back on and reading the force. Estimate uncertainties for your current measurements and your force measurements.

**Helpful Tip**

As you read the mass scale, keep in mind that there are two digits after the decimal point. See Figure 8.9(a). Also, verify that ‘g’ units are selected.

Make a plot of the force verses the current. Select the ‘2:Force’ page from the left end of the toolbar in Ga3. Fill in the ‘mass’ and current values into the table and watch the computer plot your F vs I. Verify that the force is directly proportional to the current. Do your data points lie in a straight line? Ga3 can compute force from effective mass; just enter the measured current and scale readings (in grams). Verify that the force column has units in Newtons (N) and has calculation formula

\[ 9.807 \times \frac{\text{mass}}{1000} \]

in the ‘Equation’ edit control. Estimate \( \delta m \) and \( \delta I \) and enter these into the columns provided.

**Checkpoint**

How is the magnitude of the magnetic force related to the magnitude of the current carried in a wire? How is the magnitude of the magnetic force related to the magnitude of the magnetic field? How is the magnetic force experimentally related to other measurable forces? Why did we divide \( mg \) by 1000?

Be sure your graph shows force (not mass) vertically and current horizontally. Drag a box around your data points so that all rows in your table turn grey. “Analyze/Automatic Fit.../Proportional” and “Try Fit”. This will choose the best ‘A’ to represent your data such that \( F = AI \). Make sure that the solid fit curve passes through your data points and OK. Drag the parameters box off of your data. If the uncertainty in \( A \) is not shown, right-click the parameters box and select “Fit Options.../Show Uncertainties”. This dialog box also controls the significant figures in the fitted parameters.

Once you are satisfied with your force vs. current graph, remove the wire and mass scale by reversing the process depicted in Figure 8.8. Your lab performance evaluation should suffer if you leave your station in a mess.
8.4 Analysis

Do your data indicate that your measured magnetic force is directly proportional to the current? Is this consistent with Equations (8.5) and (8.3)? Use the strategy in Section 2.9.1 to compare the slope of your force graph from Section 8.3.3 to the area under your magnetic field graph from Section 8.3.1. Can you think of any reason they should be the same? Are your units the same? Consider the uncertainty (δA) in your slope; does your slope and uncertainty bracket your area? Can you think of any errors that we have overlooked?

What does statistics say about how well your two measurements (slope vs. area) agree? Can you think of any subtle sources of error that we have not included? List as many as you can and briefly predict how each might affect your measurements. If we had figured out how to add these to our comparison, they would have made the tolerance range larger. Might any of these additional sources of error be large enough to explain why your Difference is so large?

8.5 Conclusions

What equation have we verified? Preferably reference a labeled equation in your report; otherwise, define all symbols and communicate with complete sentences. What quantities have you measured that we might need again? Don’t forget to include your units and uncertainty. If these values are prominently featured in a table, you might reference the table instead. How might we improve the experiment? Can you think of applications for any of your observations or apparatus?

Helpful Tip

Review Appendix E from time to time to refresh your memory about report contents.
8.6 Appendix: The Uncertainty in Profile Integral

The uncertainty in magnetic field strength, \( \delta B \), moves the graph of \( B(x) \) up or down. We can form an area of uncertainty by multiplying the uncertainty in \( B \) by ‘an appropriate range’ of \( x \). See Figure 8.3. The field extends to infinity; it simply gets weaker and weaker with distance. But this also indicates that we know the field to a smaller and smaller range as distance increases. Many students have guessed at this point that the ‘appropriate’ distance needed has to do with the full width at half maximum (FWHM) \( W \).

Our magnets do not generate a field that is easily modeled mathematically; however those that do suggest that \( 2W < D < 3W \) is the distance we need for this purpose. For convenience we will use

\[
\delta_1 = 2W \delta B.
\]

The uncertainty in Hall probe position, \( \delta x \), moves the left half (and the right half) of the ‘bell’ curve left or right. This forms an area of uncertainty if we multiply by the height of the bell

\[
\delta_2 = B_{\text{max}} \delta x.
\]

Each of these independent uncertainties are reduced by the fact that the area is formed by all of the data points and thus the computed area averages the uncertainties among the several points. To decide how much averaging has occurred, it is necessary to divide by ‘an appropriate number’ of points. I have probably spoiled the surprise at this point, but this will be determined by the full width at half maximum and the distance between points. We will use

\[
N = 2 \frac{W}{\Delta x}.
\]

Altogether,

\[
\delta A_I = \sqrt{\frac{\delta_1^2 + 2 \delta_2^2}{N}} = \sqrt{\left(\frac{(2W \delta B)^2 + 2(B_{\text{max}} \delta x)^2)}{2W} \frac{\Delta x}{2W}} \Delta x.
\]

(8.7)