GEOMETRIC OPTICS

Introduction

In this and the previous lab the light is viewed as a ray. A ray is a line that has an origin, but does not have an end. Light is an electromagnetic disturbance, and as such is described using Maxwell's equations, which expresses the relationship between the electric and magnetic fields in an oscillating wave.

Light propagates as a wave, yet many optical phenomena can be explained by describing light in terms of rays, which in the model for light travel in straight lines in a homogeneous medium. This model is referred to as Geometric Optics and is a very elementary theory. In this theory, light travels from its origin at a source in a straight line, unless it encounters a boundary to the medium. Beyond this boundary may be another medium which is distinguished by having a speed of light different from the original medium. In addition, light may be reflected at the boundary back into the original medium. A light ray that returns to the original medium is said to be "reflected". A ray that passes into the other medium is said to be "refracted". In most interactions of light with a boundary both reflection and refraction occur. In this lab we will observe how the laws governing refection and refraction which were explored in the previous lab can be applied to the situation of imaging with mirrors and lenses.

Any mirror or lens can be referred to as an optical element. Imaging refers to the process by which a single optical element or a set of optical elements produce an image or reproduction of the light coming from an object. We perceive an object by observing light coming from the object. We are accustom to the fact that light diverges from an object and travels in a straight line from the object. Our two eyes and our sense of perception use these facts to locate objects in our environment and create a mental picture of our surroundings.

An optical element can redirect divergent light rays to appear to diverge from a different source. Thus an image is formed apart from the original object. The image can have two natures. If the light is merely redirected to appear to diverge from a different location, and thus the light in fact does not actually diverge from that point, that is it can not be traced back to the origin of the divergence, it is said to be a virtual image. The most common example would be the reflection in a plane mirror. The image would appear to be behind the mirror surface, and yet behind the mirror may be a brick wall, hardly conducive to propagation of light. On the other hand, light may be redirected to converge at a point, after which it would diverge from that point in a similar manner to the way it originally diverged from the object. The resulting image is said to be a real image when light actually diverges from the location of the image.

The principle objective of geometric optics is to be able to determine the location of an image for certain optical elements arrange in a specific geometry. This may be accomplished two ways. One can sketch key ray paths in a scale drawing of the geometry or one can calculate the image distance and properties using a set of equations.

The Plane Mirror

The mirror is the simplest of optical elements to understand. Everyone is familiar with the imaging properties of a plane mirror. If you stand in front of a plane mirror you see your image behind the mirror. The location of the image of an object placed in front of a mirror can be diagramed knowing that the surface of the mirror reflects light with an angle of reflection equal to the incident angle.

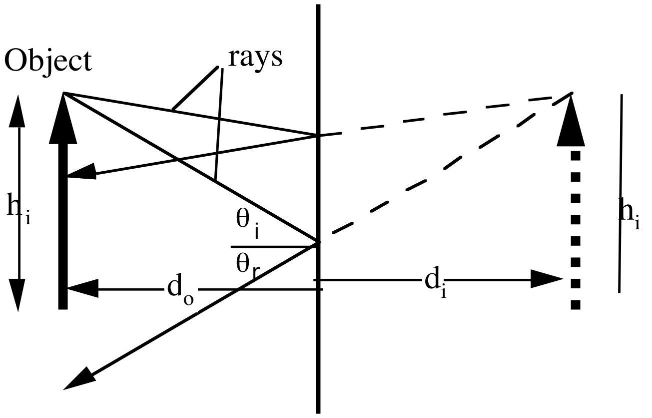

Figure 1
Ray diagram locating the image behind a plane mirror

Let us look briefly at the properties of the image formed by a mirror. It is formed behind the mirror and it is right side up with respect to the object and the same size. Note: For the image left is right and right is left. The image seems to be left - right inverted, but this is a separate issue. By convention we measure the object distance do as a positive value when in front of the mirror and image distance d_i when into the mirror as negative. Hence for the case of a plane mirror where the image is behind the mirror as much as the object is in front then

$$d_o = -d_i \qquad (1)$$

The relative size of the image compared to the object is the magnification M defined by the relation

$$M = h_i/h_o \qquad (2)$$

For a plane mirror the magnification is M = +1. We emphasize the positive sign because the sign of M has a significance we will only appreciate later. It tells us if the image is right-side-up or inverted relative to the object. Figure 1 shows a diagram locating the image of a plane mirror. In the case of a plane mirror where the image is oriented the same as the object M is positive. Even more interesting is the fact that the magnification is directly related to the relative distances the object and image are located from the mirror. In general M can be calculated from

$$M = -d_i/d_o. \qquad (3)$$

Using the relation in equation (1) we get M = +1 again. Such will not always be the case for mirrors in general.

We note finally that the rays of light form an observable image because they appear to diverge from a point other than where the object is located. For a plane mirror this point is behind the mirror. In fact, the rays cannot actually be traced back to that point. They never really pass through the point of apparent divergence. That point may very well lie inside a solid wall or behind the wall in another room. We refer to the image as a **virtual** image. Not all images are virtual. But all virtual images are erect and have negative image distances in the case of a single mirror.

The Concave Mirror

A mirror surface can be fabricated as part of a spherical surface of radius R. By convention a positive radius is on that is measured to the left of the surface assuming light to be approaching from the left to be reflected off the surface. The reflecting surface would be termed a concave mirror. The principle axis is a line that strikes the mirror in the center at normal incidence. Any light ray that is parallel to this axis when it is reflected off the mirror will arrive at the same point along the principle axis. This is called the focal point and it can be shown that this point lies a distance f from the mirror where f = R/2. These acts provide the circumstances for drawing two rays diverging from the tip of the object that will re-converge at a specific point to locate the image position for a particular object location. We show how these rays can be used to locate an object in Figure 2. A more useful interactive diagram called an applet can be found on the internet at http://www.physics.northwestern.edu/ugrad/vpl/optics/mirrors.html. The applet makes use of a third ray which if it strikes the mirror normal to the surface at zero incidence angle, is reflected back on itself. A fourth ray can be obtained where the ray strikes the mirror at the vertex, where the principle axis meets the mirror surface. Such a ray is reflected at an angle equal to the angle between the ray and the principle axis.

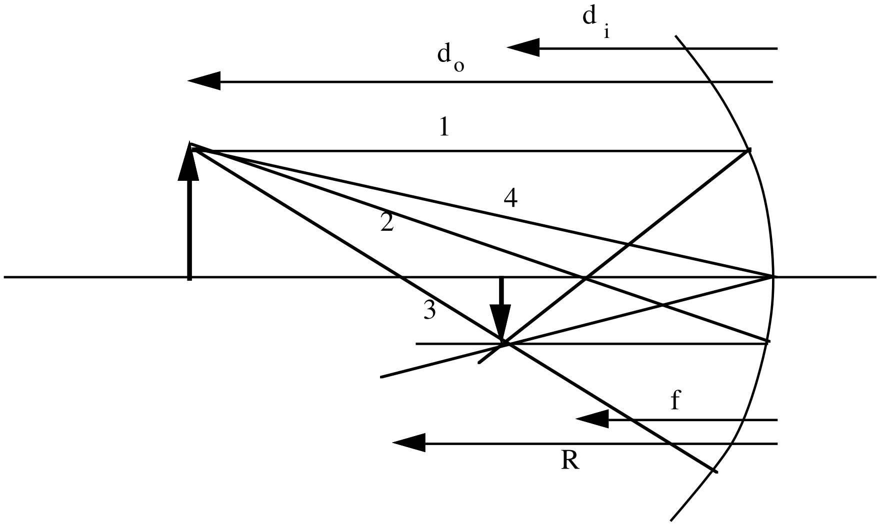

Figure 2
Ray diagram for real Image in a Concave Mirror

The most obvious thing about the image formed by this configuration is that the image is inverted. It also appears in front of the mirror. Such an image is so foreign to us that you have to really see it to appreciate it. Take a common metal spoon and hold it at arms length with the concave portion facing you. When you look into the spoon you should see your reflection and everything else behind you in the room inverted. It is harder to discern that the images are in front of the mirror. Never the less, it is the case. By convention we say that an image located in front of the mirror has a positive image distance. From the definition of magnification in equation (3) M must be negative if both d_o and d_i are positive. This agrees with the fact that the image is inverted.

The image in Figure 2 can be located analytically using the expression

$$1/d_o + 1/d_i = 1/f = 2/R \qquad (4)$$

The image is reduced in size. This further bears out the relation in equation (3) that ties the magnification to the object and image distances. Here the image lies closer to the mirror than the object and hence is smaller. We also note that the rays of light actually pass through the image location and rediverge. By convention we say that this image is a **real** image. Visibly it looks like an object is really located at that point, and the phenomena is the source of many magic tricks and optical illusions. The fact that the image is inverted and located at a positive image distance goes along with the real nature of the image. A real image will be formed if the object is located anywhere between infinity and the focus of the mirror. Real image can be projected. That is, if you were to place a white card or projection screen at the point where the image is formed, the image will appear in good focus on the screen.

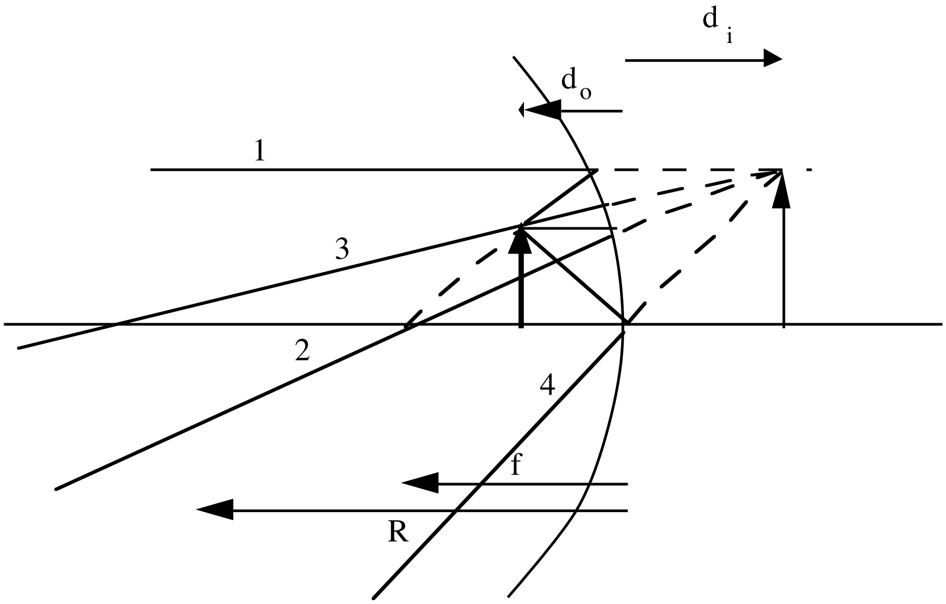

Figure 3
Ray diagram for Virtual Image in a Concave Mirror

If the object is moved to a point within the focus of the mirror the rays will no longer reconverge to form a real image. Instead the rays appear to diverge from a point behind the mirror as shown in Figure 3. Such an application would be when you use a makeup mirror. Note that the virtual image is enlarged and right-side-up. Virtual images are not projectable., since the rays never come to a focus at a real point in space.

Convex Mirror

A convex mirror is often called a divergent mirror. Light rays parallel to the principle axis are reflected away such that they appear to diverge from a focus behind the mirror. An image distance marked off behind the mirror is considered negative and locates a virtual image. In a similar manner according to the convention, a focus behind the mirror is

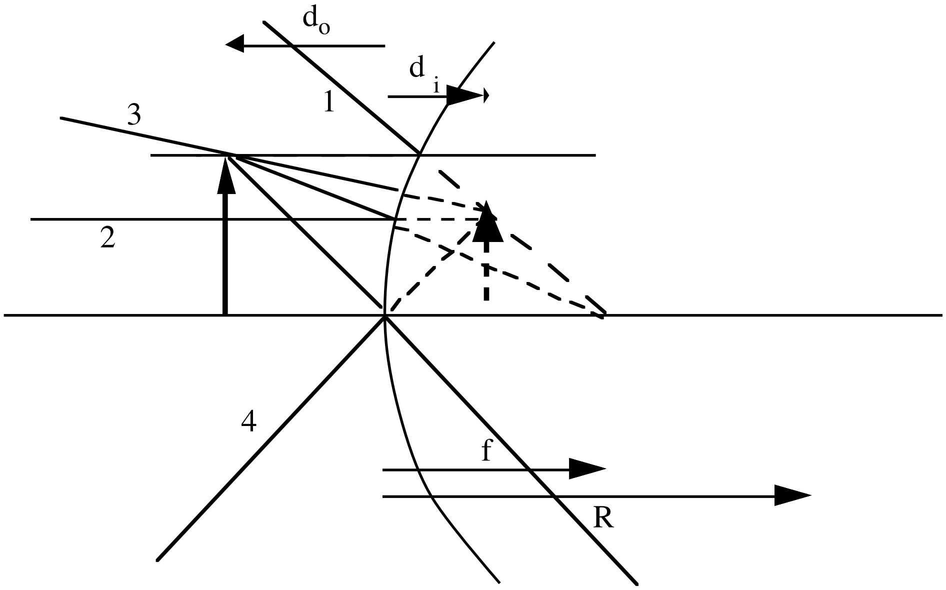

Figure 4
Ray diagram for Virtual Image in a Convex Mirror

negative, and virtual. Indeed, the rays never do pass through this focus just like rays never pass through a virtual image.

The images formed by a convex mirror are virtual regardless of where the object is placed in front to the mirror. Figure 4 shows the ray diagram locating the image. Note that ray 1 appears to diverge from the virtual focus, and ray 2 appears to be headed for the virtual focus though neither ray reaches this focus. Note also that both the radius and the focus would have numerical values with a negative sign.

Lenses

Not everyone has personal experience using lenses, or so it might seem. Besides the most common application in eyeglasses used to correct imaging problems, campers, for example, have been know to use a focusing lens to start a campfire without matches using the sun's energy. It gets more personal than that, however. Each of us uses a natural imaging system in our eye to see, which in part relies on a lens within our eye.

The surface of a piece of glass can act as an optical element that changes the direction of a light ray as it passes through the surface. We explored Snell's Law that applies to this phenomenon in the last lab. In most applications of an air-glass surface the piece of glass is thin and the rays quickly encounter the opposite side of the glass at a glass- air boundary where the light emerges from the glass. There are many applications where only this one interface is considered such as looking at objects underwater. The cornea of the eye is an example of such a surface. We will not deal with this imaging process, but consider only applications where the light emerges from the optical element into air again. Such problems are referred to as thin lens applications.

A plane piece of glass with parallel surfaces usually results in a slight sideways displacement of the original beam but no noticeable angular deflection. A plane of glass is transparent and transmits rays from an object undistorted. Such a simple case is worth a quick look compared to its reflection analog the case of a plane mirror. When looking at an object through a pane of glass we are in fact looking at the image of the object which is located at nearly exactly the same spot as the object. We have two choices. We can rewrite the imaging equations (1), (2), (3) and (4) to include the fact that the image is now on the same side of the optical element, or we can change the convention to say that a virtual image is formed on the opposite side of the optical element. The accepted convention is to choose the latter. We will see the ramifications of this in the images formed by a thin lens.

Convergent Lens

A convergent lens is one that through the action of refraction of light through the front and back surfaces of the lens causes parallel rays to come to a focus. It is the lens analog to the concave mirror. An example of a convergent lens is one with two convex surfaces although other combinations of surface curvatures can also form a convergent lens.

As in mirrors we define the principle axis is a line that strikes the lens in the center at normal incidence. Any light ray that is parallel to this axis when it is transmitted through the lens will arrive at the same point along the principle axis. This is called the focal point and it can be shown that this point lies a distance f from the lens. Unlike the mirror the radius of the surface of the lens is not easily related to the focus. Never the less two rays diverging from the tip of the object will re-converge at a specific point to locate the image position for a particular object location. One of these rays originally parallel to the principle axis is bent to pass through the positive focus on the far side of the lens. Because the lens has two operating sides, that is, it can be turned around and work just as well, lenses have a second focus on the same side as the object. Light passing through this focus is transmitted through the lens resulting in a ray parallel to the principle axis. We show how these rays can be used to locate an object in Figure 5. A more useful interactive diagram called an applet can be found on the internet at http://www.physics.northwestern.edu/ugrad/vpl/optics/lenses.html The applet makes use of a third ray that strikes the lens at the vertex, where the principle axis meets the lens surface. Such a ray passes through the lens undeflected.

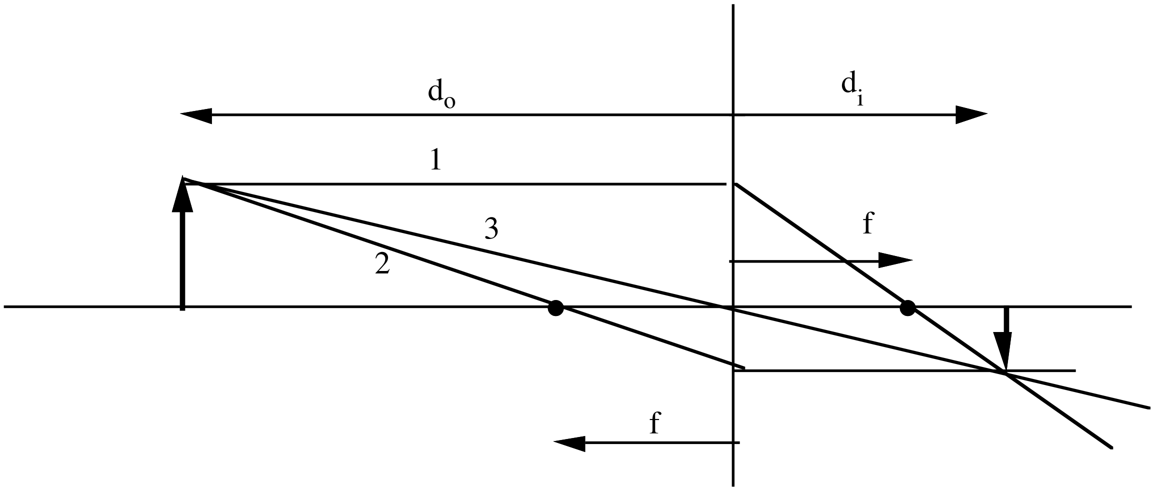

Figure 5
Ray diagram for real Image in a Convergent Lens

The most obvious thing about the image formed by this configuration is that the image is inverted just as with the concave mirror. Contrary to the mirror analog the image appears behind the lens. Such an image is so foreign to us that you have to really see it to appreciate it. Such a convergent lens is not readily available as in the case of a mirror. If you have access to a magnifying glass you can demonstrate the real image formed by such a lens. By convention we say that an image located behind the lens has a positive image distance. Note: the simple change in convention from mirrors. From the definition of magnification in equation (3) M must be negative if both d_o and d_i are positive. This agrees with the fact that the image is inverted.

The image in Figure 6 can be located analytically using the same expression we used for mirrors.

$$1/d_o + 1/d_i = 1/f$$

The image is reduced in size. This further bears out the relation in equation (3) that ties the magnification to the object and image distances. Here the image lies closer to the lens than the object and hence is smaller. We also note that the rays of light actually pass through the image location and rediverge. By convention we say that this image is a **real** image. Visibly it looks like an object is really located at that point, and a plane card placed at that point will display the image projected on it. Real images are projectable for lenses as well as mirrors. This phenomenon is applied to camera imaging as well as how our eyes work. The fact that the image is inverted and located at a positive image distance goes along with the real nature of the image. A real image will be formed if the object is located anywhere between infinity and the focus of the lens.

If the object is moved to a point within the focus of the lens the rays will no longer reconverge to form a real image. Instead the rays appear to diverge from a point in front of the lens as shown in Figure 6. Such an application would be when you use a magnifying glass. Note that the virtual image is enlarged and right-side-up. Virtual images are not projectable, since the rays never come to a focus at a real point in space.

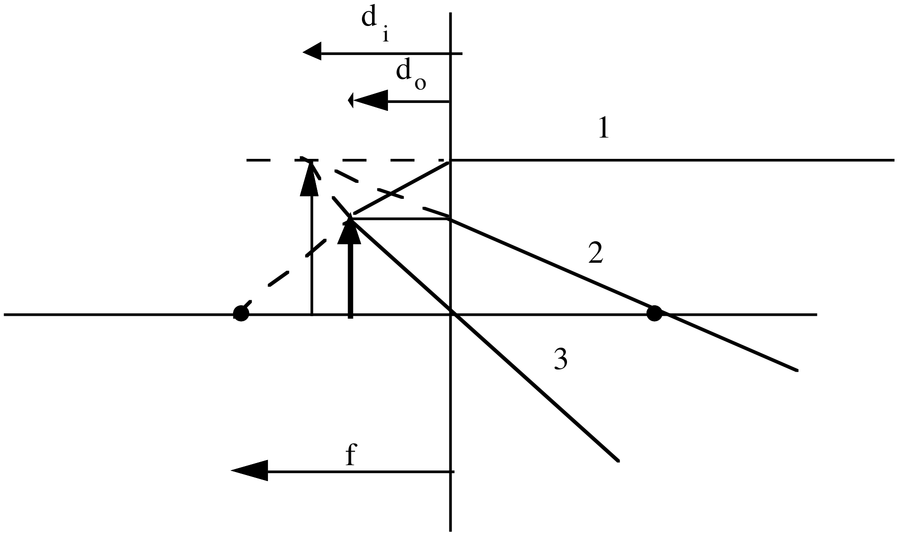

Figure 6
Ray diagram for Virtual Image in a Convergent Lens

Divergent lenses
A divergent lens is the analog to a convex mirror. Light rays parallel to the principle axis are transmitted such that they appear to diverge from a focus in front of the lens. An image distance marked off in front of the lens is considered negative and locates a virtual image. In a similar manner according to the convention, a focus in front of the lens is negative, and virtual. Indeed, the rays never do pass through this focus just like rays never pass through a virtual image. This focus is not to be confused with the real focuses of a convergent lens.

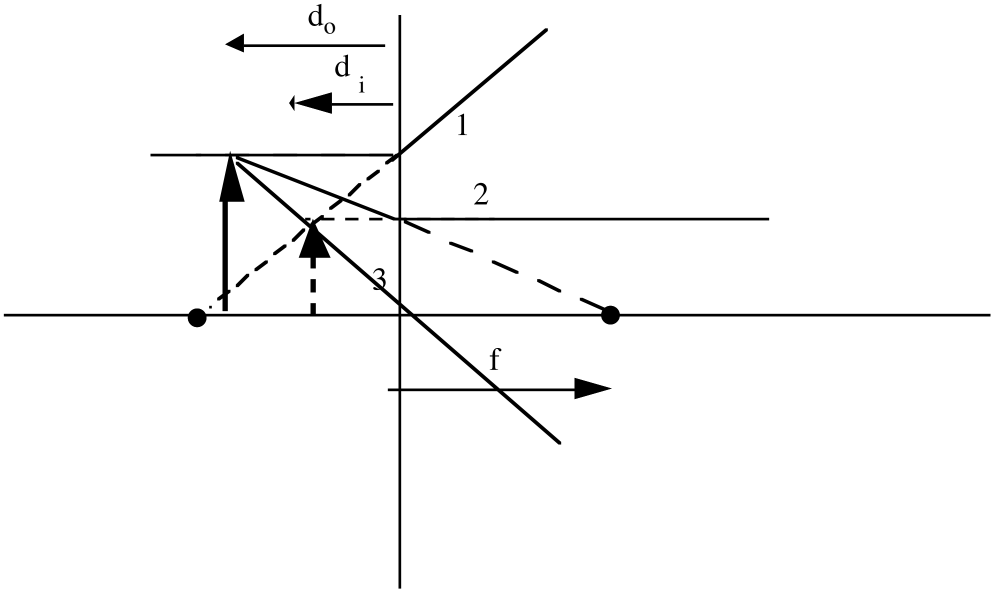

Figure 7
Ray diagram for Virtual Image in a Divergent Lens

The images formed by a divergent lens are virtual regardless of where the object is placed in front to the lens. Figure 7 shows the ray diagram locating the image. Note that ray 1 appears to diverge from the virtual focus, and ray 2 appears to be headed for the virtual focus though neither ray reaches this focus. Note also that the focus would have numerical values with a negative sign.

Experiment 1: Mirrors

Set up the ray box used in the last experiment but use the multiple (5) slit mask. The ray box may have to be adjusted to make the rays parallel. Locate the mirror block. It roughly a triangular silver plastic piece. One side is flat and the other two sides are curved, one concave and one convex.

With the flat side placed in the beam of parallel rays observe how the rays are redirected while maintaining the parallel structure. Note too that the deflection angle is twice the incident angle. A small turn of the block results in a larger change in the direction of the rays.

With the concave side placed in the beam, observe how the rays are redirected to a focal point and from there rediverge.

With the convex side facing the beam the rays diverge and appear to emanate from a focus behind the surface.

Experiment 2: Lenses

Locate the two-dimensional convergent lens (convex surfaces). Place this lens in the beam of parallel rays and observe how the rays converge to a focus on the side of the lens away from the source.

Locate the two-dimensional divergent lens (concave surfaces). Place the lens in the beam of parallel rays and observe how the rays appear to diverge from a point in front of the lens. Trace the rays with a pencil and mark the location of the lens. Then remove the lens and extend the traces to where they intersect and measure the virtual focus.

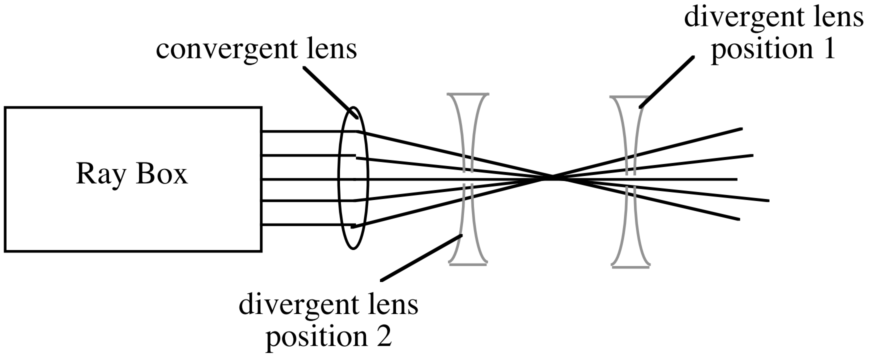

Figure 8
Placement of lenses

Place the convergent lens in the beam and form a focused beam. Use the focus as an "object". Place the divergent lens in the path of the divergent rays and observe that the rays are redirected by the lens so that they appear to be diverging from a different point. See position 1 in Figure 8. This is a virtual image. Trace the rays and mark the lens position. Remove the divergent lens and extend the rays to where they converge. Measure the virtual image distance and the object distance (from the original focused rays to the lens) and calculate the focus for this lens using equation (4). Does this focus agree with the focus measured directly?

Now place the divergent lens at position 2 in Figure 8, between the convergent lens and the focal point. Observe that the rays converge to a new focal point a little further away from the lenses. This is a real image. But where is the object? The original focus has disappeared when the lens was placed in the beam. It is a virtual object! Mark the location of the original focus, then place the lens in the beam and mark the image location (the new focus point). Again use equation (4) to calculate the focus of the lens. How well does it match the previous measurements?

Experiment 3: Lenses

Measure the focus of a real lens. Using a source far away (several meters) find the point where the image of this distant source is brought to a focus on a screen. With the room lights dimmed use the light coming through the lab door or window as an object. How does this compare with the value printed on the lens housing?

Place the optical source at one end of the optical bench and place the white screen on the opposite end of the optical bench. Place a convergent lens between the source and the screen but nearer to the source and find the spot where a focused image of the source is projected onto the screen. Observe that the image is larger than the source because the image distance is larger than the source distance to the lens. Note the position of the lens. Measure the object distance d_o and the image distance d_i and use them in the formula

$$1/d_o + 1/d_i = 1/f$$

to find the focal length of the lens. Is it comparable to the value observed above? Is it similar to the previously observed values?

Now we will measure the focus in another way. There are two possible locations where a focused image can be achieved. Move the lens closer to the screen and a second focused image will appear on the screen. Observe that this image is smaller than the object. What has changed?

Note the position of the lens for this geometry.

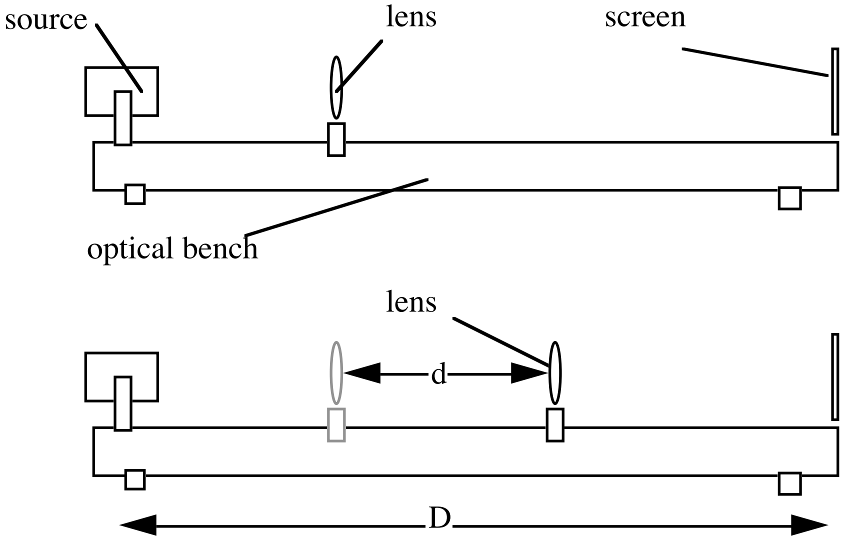

Figure 9
Measuring the focus of a lens

Calculate the distance d between the positions of the lens in each of the geometries as shown in Figure 9. This distance can be used as another way to measure the focus of the lens. Measure the total distance D between the object and the screen. The focus of the lens is calculated using

$$f = (D^2 - d^2)/4D \qquad (5)$$

Compare the two measurements. What are the limits to the uncertainties of your measurements?

Questions

Following is a list of questions intended to help you prepare for this laboratory session. If you have read and understood this write-up, you should be able to answer most of these questions. Some of these questions may be asked in a quiz preceding the lab.

- What is the difference between a real and a virtual focus?

- What is a lens?

- What is the principle axis of a mirror, of a lens?

- What is the difference between a real and a virtual image?

- What is a virtual object?

- What does the relative size of object and image depend on?

- What is the difference in the convention for mirrors and the convention for lenses?