Chapter 5

Experiment 3: Newton’s Second Law

The relationship between force and motion was first addressed by Aristotle (384 - 332 B.C.). He argued that the natural state of an object was to be at rest, and a force was not only required to put an object into motion, but a continued force was required to keep the body in motion. This may at first seem to correspond well with our everyday experiences, but it is certainly not what is taught in physics.

Galileo Galilei (1564 -1642), in addition to his postulates on uniform gravitational acceleration, proposed that a body at rest is a special case of a more general state of constant motion (i.e. constant velocity). He understood that without friction acting on a body to slow it down, it might indeed continue to move in a straight line forever. Galileo proposed that bodies remain at rest or in a state of constant motion if no force acts to change this motion. Friction is just another example of a force.

Isaac Newton (1642 -1727) formalized the relationship between force and motion in his *Principia* (published in 1687). Newton proposed that the acceleration of an object is directly proportional to the net force acting on an object and inversely proportional to the mass of the object. The Law is summarized in the vector formula $\mathbf{F} = ma$. In this laboratory, we will verify this relationship quantitatively. This law describes our understanding of the dynamics of classical mechanics.

5.1 Background: Forces, Energy, and Work

The concepts of work and energy can be derived from Newton’s Second Law in mechanics. The details of these quantities will be covered in detail in lecture and in later lab exercises. For the purposes of the laboratory, basic relevant definitions are given here.

In the case of a constant force, the physical work done by this force is defined simply. If a position changes by a displacement $\Delta x$ under a constant force $F_x$ along that direction, then the work done by the force is

$$W = F_x \Delta x = F \Delta x \cos \theta$$

(5.1)
where $\theta$ is the angle between the direction of the force and the direction of displacement. This can be succinctly written as a vector dot product:

$$W = F \cdot \Delta x$$  \hspace{1cm} (5.2)

Kinematics equations for uniform acceleration can be manipulated to obtain a useful relationship between physical work and the change in kinetic energy known as the \textbf{Work-Energy Theorem}:

$$\frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2 = \Delta KE = W = F \cdot \Delta x$$  \hspace{1cm} (5.3)

The Work-Energy Theorem is only valid when $W$ is the total work done by all forces acting on an object. Since all forces can change kinetic energy, it is important to be able to know all of the forces acting on the object under study. In particular, we often would like to eliminate friction as a relevant force since its magnitude can rarely be directly measured (often resulting in energy lost from the system, according to the Work-Energy Theorem). This may not be entirely possible, and so you should be aware that this may influence your results despite the considerable expense and effort that has been made to minimize friction in constructing the laboratory equipment.

### Historical Aside

One might take pause to appreciate the fact that Galileo and Newton were able to discover the principles of mechanics (forces, energy, etc.) without the benefit of technically advanced equipment. Galileo rolled spheres and cylinders down inclines and dropped objects from the Tower of Pisa. Newton extended Galileo’s observations to the motion of planets and moons of our solar system. Yet, they were able mentally to extract the kernel of truth from such an environment and to recognize the universality of the laws of motion.

### 5.2 Apparatus

We will be using an air track for this experiment. It consists of a hollow extruded aluminum beam with small holes drilled into the upper surface. Compressed air is pumped into the beam and released through the holes. This forms a cushion of air that supports a glider on a nearly frictionless surface.

### Helpful Tip

Do not move the air track. It is leveled and difficult to readjust.

Attached to the air track is a sonic motion sensor. The computer signals the range finder
to emit a sound pulse. The pulse reflects off the plastic card attached to the glider and returns an echo to the motion sensor. The computer receives the signal and calculates the position of the glider from the time delay between sending the pulse and receiving the echo and the known speed of sound waves in air. Computer software plots the data and can use the data to calculate velocity and acceleration. The setup is shown in Figure 5.1.

**Helpful Tip**
Do not disturb the rangefinder. Your data depends upon its proper alignment.

The computer is actually doing the same measurement you did in the first laboratory when you determined the positions of the air puck by tediously measuring the distance from a reference line to each of the dots laid down by the spark timer. The computer’s fast speed enables it to process more position data while you concentrate on the physics involved rather than the calculations. The computer is also doing the same calculations as you did when you found average velocity from displacements and time intervals.

The glider has a string attached to it which runs over a pulley at the end of the track opposite the range finder. At the other end of the string is a weight holder. Weights can be added to vary the accelerating force on the glider. The vertical force of gravity acting on the weights is transferred via the pulley to a horizontal tension applied to the glider. If friction and a few other small forces can be neglected, only this gravitational force accelerates the glider and the hanging masses at the same rate. The string’s length maintains a constant distance between the two so the time derivatives must also be the same. You can draw the force diagrams and solve Newton’s equations for the expected acceleration before coming to lab for a better understanding of this.

**Helpful Tip**
Be sure that the string touches only the glider, the round pulley, and the weight hanger; otherwise, the string will experience a large frictional force that is not included in your data analysis.

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**Figure 5.1:** Photograph of the air track, glider, weights, and motion sensor used to examine Newton’s second law of motion.
We are using Pasco’s Capstone program with their 850 Universal (computer) Interface. We have already prepared Capstone to gather your data and we have saved the setup for you to load. Open the “Newtons Law.cap” file from the lab’s website:

'http://groups.physics.northwestern.edu/lab/newtons-law.html’

Before proceeding we must verify that the track is level. Each day the tracks are pre-adjusted by the Laboratory Assistant. This is a delicate adjustment and should only rarely need to be done if the track is not moved around on the table and the table is not moved around on the floor. To test the level, momentarily detach the string and weight holder from the glider. With the glider near the center of the range of motion on the track, turn on the air supply, and verify that the glider remains (mostly) at rest when free to move along the track. Be careful that slight gusts of air from other sources are not affecting the motion of the glider. Be careful not to bump the air track or the table. If the air track appears to be out of level (noticeable and sustained acceleration), let the Teaching Assistant know before attempting any adjustments. With the permission of the Teaching Assistant you may adjust the level screws on the legs of the air track to bring the track to level.

**Helpful Tip**

Random or back-and-forth motion is not an indication of being unlevel; only continuous acceleration indicates that the track needs leveling. Even a mild breath will move the glider!

**WARNING**

Small adjustments make a big difference; since friction is so low, even a tiny component of g along the track will cause acceleration.

### 5.3 Procedure

We will hang masses under their gravitational weights to provide known external forces. This force will accelerate the air track cart. Table 5.1 shows the measured masses of these weights.
5.3.1 Acceleration vs. Unbalanced External Force

In this experiment we will measure the acceleration of the glider under conditions of varying accelerating force provided by varying the weights hanging from the pulley. Choose an initial weight for the weight holder at the end of the string. The masses of the various weights is shown in Table 5.1. You will want to use total mass combinations in the range from 2g to 22g. Start with an initial mass of 2.0g or 4.0g. Make repeated runs with at least 5 different mass combinations up to 22g.

For each run start by moving the glider away from the pulley until the weight hanger is near the pulley; hold it there. Placing a finger in contact with the air track and glider simultaneously provides enough friction to hold the glider. Be sure the software is at the point where the velocity graph is visible. To begin taking data, click the “Record” button at the bottom left and release the glider. Be sure not to obstruct the path the range finder’s sound wave needs to travel or reflections from you, the video monitor, etc. will confuse your data. If this happens merely delete the data run and retake the data. If your data is noisy, check that the reflector above the glider is perpendicular to the track and that the visual image reflected from the Motion Sensor verifies that it is pointed correctly. Ask your instructor for assistance.

When the glider hits the end of the track or the hanger hits the floor, click the “Stop” button. (The “Record” button turns into the “Stop” button when pressed and vice versa.) You should now see a plot of the glider’s position and velocity displayed as a function of time. You can choose to retake the data by deleting the data (button at the bottom right) and repeating or just leave the bad results and take new data over the old. If everything seems okay, proceed to analyze the data run. Let the mouse cursor hover on the velocity plot so that the toolbar appears above the velocity vs. time graph. Click the ‘Data Selection Tool’ (●) on the toolbar and size it to highlight the appropriate section of data points. Select the linear fit option from the curve fitting tool (●) and enable the tool. Optionally, from the position graph choose the quadratic fit after selecting the appropriate data points. A square will appear with the pertinent fit results. If necessary, right-click the parameters box and enable the “Show Uncertainties” option.

# Table 5.1: Measured masses of hanging weights and weight holder.

<table>
<thead>
<tr>
<th>Description</th>
<th>Mass (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holder</td>
<td>1.984 ± 0.035</td>
</tr>
<tr>
<td>Small Black</td>
<td>0.961 ± 0.019</td>
</tr>
<tr>
<td>Large Black</td>
<td>1.9430 ± 0.0087</td>
</tr>
<tr>
<td>Small Silver</td>
<td>4.926 ± 0.039</td>
</tr>
<tr>
<td>Large Silver</td>
<td>9.941 ± 0.045</td>
</tr>
</tbody>
</table>

Checkpoint

What uncertainties should you record for your hanger’s and weights’ masses?

Note the mass of the hanging weight \( (m \pm \delta m) \) g and the corresponding fit parameter \( (a \pm \delta a) \) m/s\(^2\) in a nice table for later use. Now change the hanging mass by adding and/or removing weights on the hanger and repeat the experiment. Repeat the experiment at least five times with at least five different hanging weights.
Since we want to check whether $F = Ma$, run Vernier Software’s Graphical Analysis 3.4 (Ga3) program. We have already prepared “Newtons Law.ga3” and stored it on the website as well

'http://groups.physics.northwestern.edu/lab/newtons-law.html'

You will need to right-click the link from the FireFox browser and to save the template (Downloads, Documents, or Box Sync/Physlabs would be good places). Next, click the blue arrow at the top right and select the “Newtons Law.ga3” file. Do your data points seem linear? If so, then it is reasonable to fit them to a line.

**Checkpoint**

What does Newton’s law $F = Ma$ predict for $F$ versus $a$ for this experiment? What do we expect for the line’s slope and $y$-intercept?

As you enter “Hanging Mass” values in the designated units, the computer enters the calculated force in the calculated column. You should check one or two of these values to make sure you entered the correct number and that you understand how the force was calculated. You can double-click the “Force” column heading to see how the title, units, and formula are entered; you can also see the formula that the computer uses.

Finally, we need to plot $F$ vs. $a$ instead of $m$ vs. $a$. If necessary, click the “Hanging Mass” on the vertical axis and select “Force” instead. Now your Ga3 file is the same as ours for Part 1; feel free to save yours frequently to avoid losing your work.

Now let us try to fit our data points to the $F = Ma$ model suggested by Newton. If your mass column is not sorted numerically, choose “Data/Sort Data Set/Force vs. Acceleration” from the menu, sort using any column. Select the data points you want to analyze by drawing a box around them with your mouse, choose “Analyze/Curve Fit/Proportional” from the menu. Click “Try Fit” and verify that the black model line passes through your data points; click “OK.” The computer will choose values for the proportionality constant ($A$) to minimize the vertical distances between your data points and the straight line given by $y = Ax$. The process the computer uses is called the “Least Squares Fit.”

**Historical Aside**

A Linear Regression is a particular least squares fit that can be solved exactly; this yields a set of equations which determine slope and intercept ($y = Mx + B$) of a straight line that best represents the set of $(x, y)$ values. These equations were derived by using calculus to find the minimum of the sum of the squares of the deviations of $y$ value of the line at each $x$-value of data from the corresponding $y$-value of data. These equations have been written into Ga3 and are used for the ‘Data/Linear Fit’ option. The solution is available on the web.
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Record the slope of the best fit line, its uncertainty, and its units in your notebook. We can represent experimental uncertainties in either of two equivalent ways. For example, Newton’s second law predicts our glider’s mass to be $M = (0.210 \pm 0.006)$ kg. We can also express the uncertainty as a percent (%) of the measurement $M = 0.210\text{kg} \pm 3\%$. If Ga3 does not show the uncertainties in slope and intercept, right-click on the parameters box, “Fit Options...”, and select the “Show Uncertainties” option. You should write the values you measured in your lab notebook. You should weigh your glider and record its mass, uncertainty, and units in your lab notebook. The value we get using Newton’s second law should be close to this value.

### Helpful Tip

The scale can measure the gliders’ masses to four significant figures; however, it can measure the 1 g, 2 g, 5 g, and 10 g masses to little more than one significant figure. You can improve this by measuring them in groups of ten and dividing the result by 10, but a far better strategy is to use the values in Table 5.1 that were obtained with a more sensitive (and more fragile) scale.

#### 5.3.2 Acceleration Proportional to Inverse of the Mass

In this experiment we will measure the acceleration of the glider with a fixed hanging weight on the weight holder at the end of the string while varying the mass of the glider. The initial mass of the glider is about 0.20 kg. You can check your glider’s mass with the electronic scale in the lab. Don’t forget to record your units. The mass of the glider can be changed by adding weights to the thin rods extending from each side and the top of the glider. The cylindrical weights which fit over these rods each have a mass of $(50.00 \pm 0.01)$ g. A maximum of four weights can be added giving you five possible mass values between 0.2000 kg and 0.4000 kg. Keep the glider balanced across the beam by adding equal weights to each side; odd weights can be added to the top.

### Checkpoint

What value of uncertainty should you record for the scale’s reading?

Obtain values for the acceleration of the glider for each of five values of glider masses using the same procedure followed in Section 5.3.1. Use a mass of 10 g on the 2.0 g weight holder for a total of $(11.925 \pm 0.057)$ g. This will produce an accelerating force of

$$mg = (0.011925\text{kg})(9.807\text{ m/s}^2) = (0.11695 \pm 0.00056)\text{ N}.$$  

You can use that data point from the first experiment, and save yourself some time. Make a table of the results. Plot the acceleration as a function of the mass of the glider.
Checkpoint
Do you get a linear relationship? If not, what does the plot suggest?

Helpful Tip
Determine a way to obtain a linear plot by rearranging Newton’s second law.

You should use the Least Squares Fitting Program (Ga3) again to find the slope of the line, the uncertainty in the slope, and the units of both. “Newtons Law.ga3” is already setup to process this data also; just select “Page 2” instead of “Page 1” from the toolbar. Alternatively, you can adapt your previous setup by renaming the columns, changing the “Equation” formula, and selecting the correct columns for your graph.

Checkpoint
What measurement corresponds to the slope of this graph?

5.3.3 (optional) Test of the Work-Energy Theorem

In this experiment we will measure the position and velocity of the glider at two separate points and compare the change in kinetic energy with the work done by the force of the string on the glider.

You may rerun the experiment for a specific set of accelerating weights and glider mass or you can use the data from the last run. Whatever the source of data, be sure that you measure the position \( x \) and the velocity \( v \) at the same times. If you move the mouse cursor to one of the graphs, you will find that a toolbar will appear above the graph. On the toolbar will be a button to “Show Coordinates and access Delta Tool” (🗖). When you push the button, a pair of dotted \((x, y)\) axes will appear at the top left of the graph. Grab the square at the ‘origin’ with the mouse and drag it to your data points. The coordinates of the data point, \((t, x)\) or \((t, v)\), will be displayed in a box. After you drop the origin, you can grab this text box and move it around with your mouse if you choose; it might be necessary to uncover the origin so that you can move the ‘origin’ to the next point you wish to study.

Select two points, one near the start of the motion of the glider and one where the glider is near the end of the track. Be sure to choose points on the constant acceleration section of the graph (and be sure the time for velocity is the same as the time for position.) Record the readings of \( x \)-position and corresponding velocity \( v \) of the glider for these two points.

Using the velocities calculate the change of kinetic energy experienced by the glider between these two points. Find the change in position by taking the difference between the
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two position measurements. Use this with the value of the accelerating force (the mass of the weight holder and weights multiplied by \( g \)) to calculate the work done on the glider by the tension of the string pulling it.

Compare the two numbers and decide if the Work-Energy Theorem has been verified. If there is a significant discrepancy, can it be explained? Good predictions (i.e. small Difference) indicates that Newton’s law works well for this purpose whereas large differences might indicate that the law has a problem, that our data has a problem, or that our assumptions are not realized by our experiment. Check your calculations if your agreement is very bad but note that we have not considered our measurement uncertainties and, thus, have no quantitative expectation for agreement.

5.4 Analysis

Using the values you recorded in Section 5.3.1, compute the difference (\( \Delta M \)) between your measurement of glider mass (using the mass scale) and that predicted by Newton’s second law from the slope of your graph. When using the mass scale, be sure to weigh all of the mass that was being accelerated by the various forces. Refer to Section 2.9.1.

Using the values you recorded in Section 5.3.2, compute the difference (\( \Delta F \)) between your measurement of applied force (from the constant hanging mass) and that predicted by Newton’s second law from the slope of your graph.

(\text{optional}) Using the values you recorded in Section 5.3.3, compute the difference (\( \Delta E \)) between the change in kinetic energy and the work done by gravity.

Are we confident that the air track was exactly level? Did we eliminate all friction? If not, where are places we might have missed? Is the glider’s mass the only mass accelerated by the hanging weight? If not, can you think of a way to estimate a compensation? Did we include the uncertainties in our mass measurements in our \( \sigma \)? Would these uncertainties increase our \( \sigma \) and make our \( \Delta \) a smaller multiple of \( \sigma \)? What total mass was moving and thus contributing to kinetic energy? What other sources of experimental error have we excluded (by choice or by accident) from our analysis?

5.5 Guidelines

Your grade will be based on two components: your in-class performance (including your lab notes) and your brief written report communicating your work and its implications.

5.5.1 Notebook

Your Lab Notebook should contain the following:

- Two tables of data collected in the lab (printed or hand written).
• A figure (graph) showing the acceleration of the cart with a fixed weight of the cart versus the various mass hanging from the string.
• A figure (graph) showing the acceleration of the cart versus the reciprocal cart mass when the hanging weight was not changed.
• (optional) A calculation and comparison of the work and change in kinetic energy.

5.5.2 Report

Your written report should address the following physics:

• Does your data support Newton’s Second Law of Motion?
• (optional) Does your data support the Work-Energy Theorem?
• Since the Work-Energy Theorem follows directly from Newton’s Second Law, what does your answer to the second part imply about the first part?
• “Yes” and “No” are terrible answers to these questions. Use these questions to guide your discussions and report structure.
• Don’t forget to label your figures and tables (including units); don’t forget to discuss each figure and each table in your text.
• Your report’s ‘Analysis’, ‘Discussion of Results’, etc., should closely follow the Analysis in your notebook as described Section 5.4. See Appendix E.
• Your report should always summarize the physics that your data supports or contradicts and all physical constants that you have measured in your ‘Conclusions’. Any suggestions for improvements to the experiment and/or applications of what you observed or used are also welcome.