Chapter 8

Experiment 6: Collisions in Two Dimensions

Last week we introduced the Principle of Conservation of Momentum and we demonstrated it experimentally in linear collisions. This week we will extend this demonstration to include two-dimensional collisions. It turns out that multi-dimensional collisions are one of our main sources of information about sub-atomic and other fundamental particles, so understanding momentum and energy conservation in these situations has broad significance to physics.

Historical Aside

Conservation of energy and momentum in collisions (and associated particle decays) are exceptionally important in the discovery and identification of new sub-atomic particles.

Figure 8.1: A photograph of the apparatus detailing relevant controls.
Historical Aside

In 1930, the neutrino was proposed by Wolfgang Pauli to account for the lack of conservation of energy and momentum in beta particle decay. The neutrino was directly detected in 1956 and observed to obey the characteristics predicted by the conservation laws (and eventually its detection received a Nobel Prize). More recently, in 2012, the Higgs boson was discovered at the Large Hadron Collider at CERN (also prompting a Nobel Prize). This apparatus collides two high-energy protons together to produce a stream of sub-atomic particles, including the Higgs, which was detected by looking at the energy of other emitted particles from the collision. Clearly the physics of sub-atomic particles are far more complex than Newton’s Laws, but the principles of conservation of energy and momentum in collisions are key ingredients for understanding the output of high-energy collisions.

In this experiment the elastic collision between two air hockey pucks is recorded. The information obtained is used to confirm the conservation of momentum in 2D and of kinetic energy in elastic collisions. As illustrated in Figure 8.1, we once again utilize the pucks and table from our first two experiments. This time the table is level, however, and we use two pucks.

8.1 Background

We wish to confirm the principle of the conservation of momentum; in other words we wish to confirm that

\[ \mathbf{p}_{\text{before}} = \mathbf{p}_{\text{after}} \]  

where the subscripts are with respect to the time of the collision. Note that in our current approach, we are considering vector momenta, which means that each component of the vector and not just the magnitude must be conserved. Figure 8.2 represents these two situations. Two vectors are equal only when their components are identical. To test Equation (8.1) we must determine and then compare the momentum components of the total system (projectile and target) before and after the collision. If the subscripts ‘p’ and ‘t’ indicate the projectile and the target pucks respectively, and if the primed quantities refer to the velocities after the collision, then Equation (8.1) can be written as follows,

\[ m_p \mathbf{v}_p + m_t \mathbf{v}_t = \mathbf{p}_p + \mathbf{p}_t = \mathbf{p}_p' + \mathbf{p}_t' = m_p \mathbf{v}_p' + m_t \mathbf{v}_t' \]  

(8.2)

We choose the velocity of the t-puck to be zero and if we define the \( \mathbf{v}_p \) to be along the \( x \)-axis; we can rewrite the last equation in components along the \( x \) and \( y \) axes,

\[ m_p \mathbf{v}_p = p_{px} + p_{tx} = p'_{px} + p'_{tx} = m_p \mathbf{v}_p' \cos \theta_p' + m_t \mathbf{v}_t' \cos \theta_t' \]  

(8.3)

\[ 0 = p_{py} + p_{ty} = p'_{py} + p'_{ty} = m_p \mathbf{v}_p' \sin \theta_p' + m_t \mathbf{v}_t' \sin \theta_t'. \]  

(8.4)
**Figure 8.2:** Sketch of two puck trajectories before and after elastic collision in 2D. The recommended $x$-axis is shown and the dashed vertical line is the recommended $y$-axis.

We will measure the distance $\Delta s_j$ each puck moves in $N_j$ time intervals. Each time interval (space between dots) is $\frac{1}{60}$ s so that $\Delta t_j = \frac{N_j}{60}$ s. From these we will calculate the velocities

$$v_j = \frac{\Delta s_j}{\Delta t_j}.$$  

(8.5)

The subscript $j$ will be projectile before, projectile after, and target after the collision—each in turn. The easiest possibility is to let $N_j = 6$ so that $\Delta t_j = 0.1$ s for all three vectors; but the $N_j$ need not be the same as long as each is the correct count for the respective $s_j$ measurement.

For the velocities after the collision, we will also measure the angles between the positive $x$-axis and the vector ($\theta'_p$ and $\theta'_t$). We will use a standard protractor for angle measurements. We will then use these angles in Equations (8.3) and (8.4).

We will also wish to determine whether the kinetic energy, $KE$, is conserved and whether consequently it is an elastic collision. We will wish to confirm that

$$\frac{1}{2} m_p v'_p^2 = KE_{\text{before}} = KE_{\text{after}} = \frac{1}{2} m_p v'^2_p + \frac{1}{2} m_t v'^2_t.$$  

(8.6)

Please notice that energy is a scalar (not a vector, such as momentum $p$ or velocity $v$) and consequently you do not need to use components.

### 8.2 Apparatus

The apparatus photographed in Figure 8.1 has been used previously in labs 1 and 2, so we will forgo most of the apparatus’ description. The two air pucks are the solid puck, which is the one used in the first and second labs, and a less massive rimmed puck. The first one is
used as a projectile \((p)\) and the second one as a target \((t)\). The table and gravity combine to constrain both pucks to move in a plane. To qualify for the Conservation of Momentum using measured velocities, we must eliminate all external force components in the plane of motion; this means leveling the table so that gravity is perpendicular to the plane of motion and reducing friction as much as possible.

### Checkpoint

What conditions must be met in order for momentum to be conserved in a three-dimensional collision?

Each of the pucks is also fitted with a pressurized air hose to push a cushion of air under the pucks to minimize friction and a pointed electrode periodically pulsed with high voltage to record the positions of the pucks at the times of these pulses. Knowledge of these positions and the times of the pulses will allow us to determine the pucks’ velocities. The masses of \(p\) and \(t\) pucks are

\[
m_p = (1.200 \pm 0.005) \text{ kg} \quad \text{and} \quad m_t = (0.680 \pm 0.005) \text{ kg}
\]

but this does *not* include the attached hoses and wires.

### WARNING

Do not turn the air supply on all the way. Only turn it on a little way. The puck will float well. Do not drop the puck. The table top is glass!

### WARNING

The puck should be on the white record paper and not on the carbon paper whenever the push-button activating the pulse generator is pressed. The white paper should be on the carbon paper. If you touch the high voltage terminal and ground at the same time, you may get a shock — harmless, but unpleasant! Always make sure that the ground clip is properly connected to the carbon sheet before activating the pulser.

### 8.3 Procedure

Verify that the air table is level, first with the level and then by trying to keep the floating \(t\)-puck stationary at the center of the air table. It will also help to lift the hose vertically with only a little slack. With the \(t\)-puck at rest near the center of the table, practice making a two-dimensional collision in which the projectile puck is given a velocity of about 0.5 m/s
and the angle between the two pucks, after the collision, is about 30°-60°. Now make a
record of the collision by holding the spark timer button throughout the relevant motion.
Be sure to note whether the projectile is the heavy or the light puck in your Data. Also note
both masses, units, and uncertainties.

Draw arrows on the record paper that show the directions of the pucks’ velocities before
and after the collision. Since the target puck was not moving prior to the collision (hopefully),
only three vectors will be shown. If this is done correctly, the projectile puck will be moving
inward prior to the collision and both pucks will be moving outward after the collision.

Remove the record paper from the apparatus and transfer your velocities and axes to the
back side where the puck locations are recorded. Note that the puck trajectories are usually
not perfectly straight. Using the straight edge, carefully extend the incoming velocity across
the entire page and label it to be the \( x \)-axis. We desire the velocity vectors immediately prior
to and immediately following the collision; however, to get accurate speeds we must measure
reasonably long distances (\( \Delta s \geq 3 \) cm) so that our measurements (signal) are significantly
larger than the uncertainties (noise). Using a straight edge, draw lines along the remaining
velocities as close to the instant of collision as you can guess. These lines will need to extend
about six inches on each side of the collision and they will need to extend back until they
cross the \( x \)-axis. Draw arrows on the correct ends of these lines to indicate the correct
orientation of the velocity vectors. If necessary, check your original notes on the front side
of the paper to make sure you get these right.

Referring to the front side of the page, construct the \( y \)-axis perpendicular to this \( x \)-axis. Since we are working on the back of the page, the \( y \)-axis direction and our angles will be
opposite to what we normally experience; positive angles will now be clockwise from \(+x\) and
negative angles will now be counterclockwise from \(+x\). Use the protractor to measure the
angles of the outgoing velocity vectors and record them in your notebook. The angles you
want to measure have the tips of the vector arrows out by the protractor scale and the tail of
the vector arrows crossing the \( x \)-axis. Don’t forget to record the units and the uncertainties
of these angles in a table.

**Checkpoint**

How accurately can you construct these vectors? How accurately can you position and
read the protractor? Let these questions guide you when estimating the experimental
uncertainties in your angle measurements.

For each of the three vectors, we now want to measure the puck’s speed \( v_j \). Beginning
near the collision, measure the distance \( s_j \) between two dots at least 3 cm apart. Make sure
both dots are on the same trajectory; if there is doubt, begin further from the collision.
Indicate the terminal dots on the record paper and record the distance between them, the
distance’s uncertainty, and units in your notebook. It would be best to organize these data
in tables. Repeat for all three (four?) velocity vectors.
Now count the number of spaces, \( N_j \), between the two measured endpoints in each vector. Be very careful to correlate each \( N_j \) with its respective \( s_j \); if you mix them, up your velocities will mean nothing. Compute all of the \( \Delta t_j = \frac{N_j}{60} \) s and record them in your data table. Once in a while the sparker skips a dot; you must count this skipped dot to obtain the correct \( \Delta t_j \); circle the place you think the dot should be. Use Equation (8.5) to compute the puck speeds along each of the trajectories. Don’t forget to estimate your measurement uncertainties and to record your units. If your target trajectory before the collision was not a point, this motion will be a substantial source of error. (It is possible to compensate for this, but it is subtle and tricky.)

Use your measurements to compute the \( x \) and \( y \) components of each of the three (four?) momenta:

\[
\begin{align*}
p_{px} &= m_p v_p, & p_{tx} &= m_t v_t \cos \theta_t \quad \Rightarrow 0 \\
p'_{px} &= m_p v'_p \cos \theta'_p, & p'_{tx} &= m_t v'_t \cos \theta'_t \\
p_{py} &= 0, & p_{ty} &= m_t v_t \sin \theta_t \quad \Rightarrow 0 \\
p'_{py} &= m_p v'_p \sin \theta'_p, & p'_{ty} &= m_t v'_t \sin \theta'_t
\end{align*}
\]

and estimate their uncertainties using the appropriate uncertainty equations. We have provided an Excel® worksheet to propagate the uncertainties ‘2D Momentum.xlsx’ at

'http://groups.physics.northwestern.edu/lab/’.

This is necessary since none of the formulas (Equations (2.4)-(2.8)) accommodate trigonometric functions. The student may also enter formulas and allow the spreadsheet to perform his calculations of momentum components or he can simply enter the correct numbers. The results may be printed or copied to a Word® document for inclusion in the report.

**Checkpoint**

If these calculations were done correctly, all of the \( x \) components (except possibly the initial target momentum) should be positive and the \( y \) components should have opposite signs. This is insufficient to assure correctness but might indicate simple mistakes.

Find the total \( x \) momentum for the two masses and their uncertainties using

\[
\begin{align*}
p_x &= p_{px} + p_{tx}, & p'_x &= p'_{px} + p'_{tx} \\
p_y &= p_{py} + p_{ty}, & p'_y &= p'_{py} + p'_{ty}
\end{align*}
\]
Helpful Tip

Remember how to find the uncertainty of a difference: you *add* the uncertainties of the subtracted terms in quadrature (square, add, take the square root).

Find the initial and final kinetic energies using

\[ KE = \frac{1}{2} m_p v_p^2 + \frac{1}{2} m_t v_t^2 \]

\[ KE' = \frac{1}{2} m_p v_p'^2 + \frac{1}{2} m_t v_t'^2 \]

and find their uncertainties. If your target was initially at rest, then \( v_t = 0 \) and you need to omit the uncertainties in \( m_t \) and \( v_t \) from the initial energy uncertainty calculation.

### 8.4 Analysis

Determining the uncertainties \( \delta p_j \) in the components of momentum \( p_j \) due to the uncertainties \( \delta \theta_j \) in the measured angles is complicated by the fact that the angles \( \theta_j \) are arguments of trigonometric functions \( \sin \theta_j \) and \( \cos \theta_j \). These possibilities were not discussed in Chapter 2. Each instructor should announce which one of the following options his/her students should pursue:

**OPTION 1:** One possibility is to neglect the \( \delta p_j \) the way we normally do not explicitly consider ‘other sources of error.’ This will grossly underestimate the \( \delta p_j \)’s and will make it less likely for our before and after numbers to agree. Since deciding where to draw the outgoing velocity vectors is our largest source of error, we do not like this option.

**OPTION 2:** A second possibility is to pretend that the uncertainties in \( \sin(\theta_j) \) and \( \cos(\theta_j) \) are equal to \( \delta \theta_j \) (in radians)

\[ \delta(\sin(\theta_j)) = \delta(\cos(\theta_j)) = \delta \theta_j \quad \text{(in radians)}. \]

This option will *overestimate* the \( \delta p_j \)’s and will make it more likely that our before momentum components will agree with our after momentum components — even in cases when they really shouldn’t; thus we also do not like this option.

**OPTION 3:** A third possibility is to use the Excel® worksheet to determine the correct \( \delta p_j \). This will alleviate the biases inherent in the above options, but provides the students with no intuition of how \( \delta \theta_j \) affects the respective \( \delta p_j \). This option is also useless if the student later faces a similar problem in his/her professional career. So we also do not like this option.

**OPTION 4:** Since every component depends upon mass, speed, and angle, we should expect that the vertical components \( p_y \) will disagree to the same extent as the horizontal components \( p_x \). This suggests a final possibility:
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1. Let the uncertainty in the change in horizontal momentum \( \delta p_x = \sum p_x' - \sum p_x \) be the observed change in vertical momentum.

2. Decide whether \( \sum p_x' \) agrees with \( \sum p_x \pm \delta p_x \).

Can you say something about the frictional forces in the air puck system? Why do the three velocity vectors not intersect at a point? Use the strategy in Section 2.9.1 to decide whether 1) initial and final \( x \) momentum, 2) initial and final \( y \) momentum, and 3) the initial and final kinetic energy agree or disagree.

What external forces can you think of that might have affected the momentum of one or both pucks? What forces (internal or external) can you think of that might have performed work on one or both pucks? Can you think of any evidence to support this? What other forms of energy might the initial kinetic energy have become? What other sources of experimental error can you think of that we have not incorporated into our total errors? Is it likely in your estimation that these considerations are large enough to explain the differences between the theories of momentum and energy conservation and your data? How might sliding friction between the pucks (an internal force) affect: 1) the intersections of the velocity lines, 2) the conservation of momentum, 3) the conservation of kinetic energy, and 4) the rotation (and the rotational kinetic energy) of the disks?

8.5 Report Guidelines

Is \( p_{\text{before}} = p_{\text{after}} \)? How well can you make this comparison? (what is the uncertainty?) Did you verify all 3D components? How might we apply this to alpha rays interacting with a gold nucleus? Was kinetic energy conserved?

Your Lab Notebook should contain the following:

- A sketch of the observed collision.
- A table of puck masses, dot distances, and trajectory angles, etc.
- Calculation of momentum (both components).
- Table of 6-8 before and after momentum components; total momentum components before and after.
- A comparison of \((x, y)\) momentum components.
- Kinetic energy (total) before and after.
- A comparison of the energies and their error analysis.
- Observations regarding error sources, external forces (impulses), other forms of energy excited by the collision.

Your Report should contain the following:

- A complete description of the apparatus and method (or a suitable reference to previous work and a description of similarities and differences).
• A schematic sketch of your collision.

• Data tables showing your raw data, your momentum components, and comparisons between initial and final momentum components.

• Discussion introducing raw data and leading to total momentum components and kinetic energy.

• Data table summarizing initial and final kinetic energies and their comparisons; can be appended to previous table.

• Discussion of error sources, external forces, work, etc.

• A statement of conclusion telling what physics your data supports, contradicts, or neither.

• Possible improvements; applications for what you have observed.